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ON THE OPTICAL CONDITIONS REQUIRED TO SECURE MAXIMUM ACCURACY OF MEASUREMENT IN THE USE OF THE TELESCOPE AND SPECTROSCOPE.

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PRELIMINARY NOTE.—The main results of the present paper were obtained in 1897 and were informally presented before the Astronomical and Astrophysical Conference at the dedication of the Yerkes Observatory in October of that year.[1] The general investigation on the "Conditions of Maximum Efficiency in Astrophotographic Work," of which these formed a part,[2] was interrupted by my departure from Yerkes Observatory in the winter of 1897–8, and since then my attention has been so continuously devoted to technical and engineering work that until very recently I have had no opportunity to put any of the results obtained at that time in form for publication. This will explain, I trust, the delay in the resumption and continuation of work begun more than five years ago.

IN the use of any optical instrument there are three quantities which are more or less closely related and which together determine what is ordinarily termed "optical definition." These

[1] *Yerkes Observatory Bulletin* No. 2; ASTROPHYSICAL JOURNAL, **6**, 150, 1897. See also papers "General Theory of Telescopic Images," *ibid.*, pp. 123, 127; and "Effect of Atmospheric Aberration on the Intensity of Telescopic Images," *ibid.*, 7, 70.

[2] See Note on the Result Concerning Diffraction Phenomena, *M. N.*, **58**, 287 (*b*).

quantities are "resolution," "accuracy," and "contrast," and the corresponding characteristics of the instrument with reference to them have been termed "resolving power," "metrological power," and "discerning or delineating power." They all depend directly on the form and distribution of intensity in the physical image of the object under examination as formed at the focal plane of the instrument, and differ from each other only in the way in which this form and distribution affect the particular use to which the instrument is to be applied. Thus in the discovery of double stars and general spectrum analysis, resolution is of most importance; in meridian circle and heliometric work and in determinations of absolute and relative wavelengths, accuracy is the first consideration; while in the study of planetary detail and what may be termed pictorial and chart photography, contrast is the quality to be chiefly considered.

The question of the resolving power of instruments has been considered for a number of general and special cases[1] by different writers. The theory of metrological power and contrast[2] has received much less attention. In the present paper it is proposed to investigate more fully the general conditions of metrological power and accuracy in the optical measurement of the relative or absolute position of points or lines.[3]

[1] RAYLEIGH, "Wave Theory," *Enc. Brit.*, **24**, §§ 11, 12, 13, and 14. "Investigations in optics with reference to the spectroscope," *Phil. Mag.*, **8**, **9**, 1879–80; with reference to the microscope, *ibid.*, **42**, 167, 1896; MICHELSON, *Phil. Mag.*, **31**, 388, 1891; **34**, 280, 1892; ASTROPHYSICAL JOURNAL, **1**, 1, 1895; WADSWORTH, *ibid.*, 52, 1895; **3**, 176 and 321; **4**, 54, 1896; **6**, 27, 1897; *Mem. Spet. Ital.*, **26**, 2, 1897; *Phil. Mag.*, **43**, 317, 1897; ASTROPHYSICAL JOURNAL, **16**, 1, 1902.

[2] The investigation of the general subject of delineating power and contrast has been begun (ASTROPHYSICAL JOURNAL, **6**, 119, 1897; **7**, 70 and 77, 1898) and the results applied in detail to some special cases. ("Astronomical Photography," *A. N.*, **144**, 97; "Photography of Planetary Surfaces," *Observatory*, **20**, 333, 365, 404; "Visibility of Linear Markings on Planets," *A. J.*, **18**, 41.) The concluding part of the general paper (ASTROPHYSICAL JOURNAL, **7**, 70) and some other papers on additional special cases were in course of preparation when the work was interrupted as explained in the preliminary note. They will be taken up again as soon as possible.

[3] The case of the measurement of position of sources of considerable angular magnitude, such as the Sun, Moon, and major planets, involve different conditions of measurement, and will for that reason be considered separately. [See *M. N.*, **58**, 288 (h).]

Let us denote the resolution of an optical instrument by R and its accuracy of measurement by A. These quantities are the reciprocals respectively of a, the limiting resolving power, *i. e.*, the angular distance between two points or lines that can just be resolved; and of ϵ, the limiting metrological power, or the smallest angular distance that can be measured with certainty. These four quantities are connected by the general relations[1]

$$\left.\begin{aligned} A &= aR, \\ \epsilon &= \frac{1}{a}\alpha \end{aligned}\right\} \qquad (1)$$

where a is a factor whose value is not constant but varies with the conditions of measurement. As has already been pointed out, a fundamental condition for attaining a maximum value of a is that the *scale* of the diffraction image of the source whose position is to be measured shall be *large* compared with the width of the reference cross-wires or points to which its position is referred.[2]

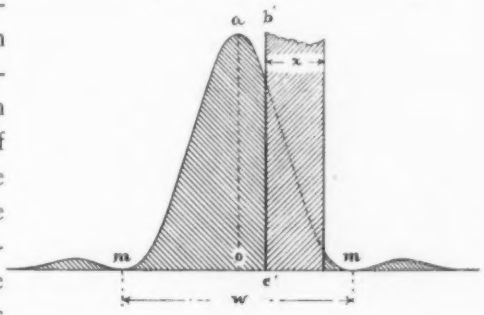


FIG. 1.

When the diffraction image is sufficiently broad with reference to the wire, x, as in Fig. 1, it is possible to locate the position of the former with reference to the latter with an error not exceeding one hundredth of the total width $mm = w$ of the diffraction image. Since the resolving power a for fine lines and points is (for rectangular aperture) equal to $\frac{1}{2}w$, it would follow that the maximum value of a might be as large as 50. Usually this degree of magnification of the image is not attainable, and the corresponding value of a is reduced. Experience, however, shows that under best conditions a value of from 10 to 15 can be attained. Thus, with a telescope having an aperture of 4 cm, the limiting resolving power of which is

[1] MICHELSON, "Measurement by Light Waves," *Am. Jour.*, **39**, 115.

[2] *Phil. Mag.*, **44**, 83; *Phys. Rev.*, **4**, 96.

about 3″, it is possible under favorable conditions to measure angular differences of position as small as 0″.20 either micrometrically or heliometrically.[1] About the same order of accuracy is attained in setting on the images of spectral lines. Thus Jewell finds that his probable error of a single setting on a line in the solar spectrum obtained with Rowland's concave gratings is about 0.001 tenth-meter.[2] The limit of resolving power of the gratings is about 0.015 tenth-meters.[3] Similarly both Campbell and Frost find that their average errors of setting on the image of a line in the star spectra obtained with the Mills and Bruce spectrographs is about 0.0004 mm and 0.0003 mm respectively.[3] With the cameras of 406 and 449 mm focal length these linear errors correspond to an accuracy in angular measurement of about 0″.19 and 0″.14. The angular resolving power of the two camera objectives (apertures 37.4 mm and 51 mm) are only 2″.5 and 1″.8 respectively for the photographic region of the spectrum, $\lambda = 4500$.

In each of the above cases the accuracy of setting (on the image of the point or line) is about fifteen times the resolution obtainable with the instrument. For this class of measurements (*i. e.*, telescopic) we may therefore assume

$$\begin{aligned} a_{\max.} &\cong 15 \\ \epsilon &\cong 0.07a \ . \end{aligned} \qquad (2)$$

In order that these values of a and ϵ may represent a real and corresponding degree of accuracy in the determination of the position of the source itself (as distinguished from the position of the image) another very important condition is necessary. This is that the distribution in intensity in the diffraction pattern *shall be symmetrical about the position of the geometrical image of the source*. For if the distribution is unsymmetrical, as in Fig. 2, the tendency will be to place the measuring wires to

[1] *Jour. Franklin Inst.*, **138**, 1; July 1894.

[2] ROWLAND, *A. and A.*, **12**, 321, 1893. [3] *Phil. Mag.*, **43**, 320.

[3] From unpublished observations communicated by these observers. In each case the average error is that of the mean of four settings only. On the best solar plates Campbell states that his average error of setting is less than one-third of this, *i. e.*, only about 0.00012 mm.

one side of the position of the geometrical image, which is at *ao*, by an amount *oo'*, depending on the excess of illumination *amm'* on that side. The error thus introduced will correspondingly reduce the accuracy of setting, so that in general we shall have to write

$$a' = ka_{\max.},$$

where k is less than unity by an amount depending on the asymmetry in the image. It becomes necessary, therefore, to investigate the various causes of such asymmetry and the effect produced by each.

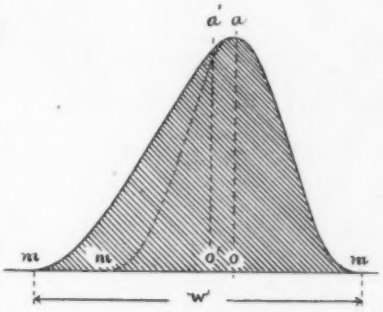


FIG. 2.

The general expression for the intensity, I^2, at any point, p, in the focal plane image is of the form[1]

$$I^2 = \left[\int\int \frac{1}{\lambda\rho} \sin \frac{2\pi}{\lambda}(at - \rho)\, i\, dx\, dy\right]^2, \qquad (3)$$

where ρ is the distance of the point p from the element of the wave-front, $dx\,dy$, whose amplitude of vibration is i. The integration is extended over the whole of the aperture through which the wave-front passes.

The expressions for the distribution in intensity in the image of a fine point which have been usually employed in discussing questions of resolving power are

$$I^2 = \frac{4}{\lambda^2 f^2}\left[\int_{-R}^{+R} \sqrt{R^2 - x^2} \cos \frac{2\pi\xi}{\lambda f}\, x\, dx\right]^2 = \frac{\pi^2 R^2}{\lambda^2 f^2} \cdot \frac{4 J_1^2\left(\frac{2\pi R\xi}{\lambda f}\right)}{\left(\frac{2\pi R\xi}{\lambda f}\right)^2} \qquad (4)$$

for a telescope with a circular aperture of radius R, and

$$I^2 = \frac{d^2}{\lambda^2 f^2}\left[\int_{-\frac{b}{2}}^{+\frac{b}{2}} \cos \frac{2\pi\xi}{\lambda f}\, x\, dx\right]^2 = \frac{d^2 b^2}{\lambda^2 f^2} \cdot \frac{\sin^2\left(\frac{\pi b\xi}{\lambda f}\right)}{\left(\frac{\pi b\xi}{\lambda f}\right)^2} = A \frac{\sin^2 \frac{\pi}{a_o} a}{\left(\frac{\pi}{a_o} a\right)^2} \qquad (5)$$

for a telescope of rectangular aperture of width d and length b. These expressions represent respectively the distribution in

[1] RAYLEIGH, "Wave Theory," *Enc. Brit.*, **24**, § 11; *Pop. Ast.*, **5**, 534.

intensity at any distance $a = \frac{\xi}{f}$ from the center of the image along a line parallel to the length b of the diffracting aperture.

In deriving the expressions (4) and (5) from (3) the following assumptions are made:

1. That the amplitude of vibration i is uniform and constant over the entire wave-front within the diffracting aperture, and that the latter is symmetrical with respect to the line joining the geometrical centers of the source and the image, *i. e.*, with the line of collimation of the telescope.

2. That the wave-front passing the diffracting aperture is truly spherical and has its center at the center of the geometrical image. This amounts to the condition that there is no aberration.

3. That the wave-length λ is constant within the limits of integration, *i. e.*, either the light is strictly monochromatic or the telescope is strictly achromatic.

In practice not one of these assumptions is strictly correct and the expressions (4) and (5) are not therefore strictly accurate. In dealing with questions of resolving power the effect of variations from these theoretical conditions has been investigated in a number of cases and has generally been found to be small.[1] We cannot, however, assume that the same conclusion holds when we come to deal with questions involving the metrological power and accuracy, for we then have to consider quantities of a much smaller order of magnitude, and an amount of disturbance or imperfection, particularly asymmetry, in the image which is negligible in questions involving resolving power (or in many cases of contrast) will introduce an error of measurement considerably larger than the limit of accuracy ϵ attainable under the best conditions.

The various causes which produce distortion and asymmetry in the diffraction image may be divided into two general classes:

[1] For special cases in which the effect is of considerable importance see "Theory of the Objective Spectroscope," ASTROPHYSICAL JOURNAL, 4, 54; "Resolving Power of Telescopes and Spectroscopes for Lines of Finite Width," *Phil. Mag.*, 43, 317; also paper, "The Effect of Absorption on the Resolving Power of Prism Trains," to be published in the February number of the *Phil. Mag.*

(A) Those of a physical nature of a character already indicated (1), (2), and (3) (p. 272).

(B) Those of a more purely instrumental nature depending on peculiarities of form or size or operation of the instrument itself.

In considering both of these classes we shall in this general paper assume the following conditions, which may nearly always be fulfilled or at least closely approximated:

(*a*) That the aperture of the instrument is always rectangular and that one of the sides is parallel to the axis or line of measurement, this axis being likewise the line along which the distribution in intensity in the image has to be considered. The assumption of rectangular aperture gives in general simpler analytical expressions, and the results obtained, in most cases, lead to the same conclusions as would be reached if circular apertures were assumed.[1]

(*b*) That in addition to being parallel to one side of the rectangular aperture the axis of measurement is so chosen that the disturbing cause is symmetrical in respect to this axis. Specific cases in which this condition cannot be fulfilled will be treated separately.

Case A (1)—Illumination over the incident wave-front unsymmetrical.

The general expression (3) may be reduced to the form

$$I^2 = \frac{1}{\lambda^2 f^2}\left[\int\int \cos\frac{2\pi}{\lambda f}(\xi x + \eta y)\, i\, dx\, dy\right]^2 + \frac{1}{\lambda^2 f^2}\left[\int\int \sin\frac{2\pi}{\lambda f}(\xi x + \eta y)\, i\, dx\, dy\right]^2. \qquad (6)$$

Under the conditions (*a*) and (*b*) assumed above we obtain in this case for the intensity of the diffraction pattern at any point on the axis ξ, ($\eta = 0$), the expressions

$$I_1^2 = \frac{d^2}{\lambda^2 f^2}\left\{\left[\int f(x)\cos\frac{2\pi\xi}{\lambda f}x\, dx\right]^2 + \left[\int f(x)\sin\frac{2\pi\xi}{\lambda f}x\, dx\right]^2\right\} = A(c^2 + s^2), \qquad (7)$$

where $f(x) = i$ represents the amplitude of vibration at different portions of the wave-front.

[1] The case of circular apertures is taken up in detail in some special cases where the results differ appreciably from those obtained with rectangular apertures.

The only case of importance of this kind is that in which the wave-front, before reaching the diffracting aperture, has traversed an absorbing medium of varying density or varying thickness as in the case of the prism spectroscope. This latter case has been recently examined in connection with the question of resolving

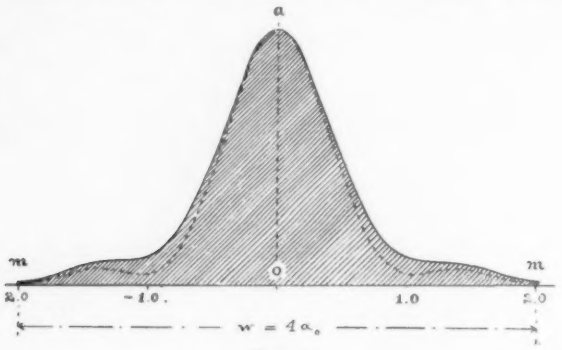


FIG. 3.

power of prism trains composed of very large or very dense prisms.[1]

The expression for I^2 which was found was

$$I^2 = \frac{d^2}{\lambda^2 f^2}\left[\frac{e^{Bb} + e^{-Bb} - 2\cos\frac{2\pi\xi b}{\lambda f}}{B^2 + \frac{2\pi\xi}{\lambda f}}\right], \qquad (8)$$

where B is a constant depending on the coefficient of absorption of the glass composing the prism train, and is determined from the relation

$$i = i_0 e^{-Bx}, \qquad (9)$$

i_0 being the intensity of light transmitted through the axis of the prism system.

The distribution in intensity represented by (8) is shown in Fig. 3 for the particular values of $B = \frac{1.386}{b}$ and $B = \frac{2.197}{b}$. For these values of B the intensity of the transmitted beam falls off 75 per cent. and 89 per cent., respectively, from one edge of the aperture to the other.

[1] "On the Effect of Absorption on the Resolving Power of Prism Trains," see footnote above.

The curve of intensities is symmetrical with respect to $\xi=0$, the position of the geometrical image. The unsymmetrical absorption therefore does not introduce any direct error of measurement at the focal plane.[1] In fact since the effect is in general to increase the apparent width w of the line, the accuracy of setting will, if anything, be slightly increased.

It may be similarly proved that an asymmetry in the form of the aperture itself will not affect the symmetry of the image about the geometrical center. Indeed, we may choose particular forms of aperture (other than rectangular) which will somewhat increase the resolving power and the accuracy of the measurement.[2]

CASE A (2).

Effect of asymmetry in the wave-front (aberration).—When the wave-front which forms the image is truly spherical its equation is

$$x^2+y^2+z^2=f^2\ , \tag{10}$$

and the distance ρ from any point xy in this front from a point $\xi\eta$ in the focal plane is

$$\left.\begin{aligned}\rho &= \sqrt{f^2-2x\xi-2y\eta+\xi^2+\eta^2}\\ &\cong f\left[1-\frac{x\xi+y\eta}{f^2}\right].\end{aligned}\right\} \tag{11}$$

If the wave-front is not spherical we may express the coördinates of the new surface with respect to the old as follows

$$\left.\begin{aligned}x' &= x+\beta x^2+\gamma x^3+\delta x^4+\cdots\\ y' &= y+\beta_1 y^2+\gamma_1 y^3+\delta_1 y^4+\cdots\\ z' &= z\ .\end{aligned}\right\} \tag{12}$$

The expression for the intensity I_1^2 at any point ξ, η, in the focal image will be found by substituting these new values for x', y', z', in (11) and (6), and performing the indicated integrations. But as before we considerably simplify the integrals with

[1] The effect of asymmetrical illumination on the displacement of an image out of the focal plane is considered under *B*.

[2] Discussion of this question will be taken up in a paper "On the Form of Telescope Aperture Best Adapted to the Resolution and Measurement of Close Double Stars," which is now in course of preparation.

which we have to deal by the assumption of conditions (*a*) and (*b*), p. 273. Then all terms in odd powers of y higher than the first disappear. Next, by changing f, equation (11), to a new value f^1 (*i. e.*, by readjusting the focus) we may cause the terms in x^2 and y^2 to disappear. Finally, since any disturbance which we need to consider is exceedingly small (as will appear later), terms in x^3 are relatively of much more importance than terms in x^4, y^4, or any higher powers, and all of the latter may therefore be neglected. Besides this any aberrational effect due to terms in x^4 will be symmetrical in its nature and will not therefore affect the *position of the center of area* of the diffraction image. The same would also be true of terms in x^3 and y^3 whose effect, however, can, as already seen, be rendered negligible by refocusing.

When we consider only terms in x^3 the expression for I^2 (6) (since sin x^3 like sin x is an uneven function), becomes for rectangular aperture

$$I_2^2 = A' \int_{-\frac{b}{2}}^{+\frac{b}{2}} \cos \frac{2\pi\xi}{\lambda f} [x + \gamma x^3]\, dx \ . \qquad (13)$$

This integral was first investigated by Airy [1] and subsequently more completely by Lord Rayleigh.[2] The latter has adapted Airy's results to the consideration of special cases for which the distortion of the wave-surface amounts to ¼, ½, and ¾ of a wave-length.

The values of I_1^2 for the value of γx^3 such that the extreme displacement of the wave-surface at the edge of the aperture is ¼ of a wave-length from the true spherical surface has been calculated from Airy's and Rayleigh's results and is given in column 2, Table I. The values of I^2 (from 5) are given in column 3, for comparison. The abscissæ are expressed in terms of a_0, the resolving power of the telescope of aperture b.

The form and relative position of the two curves I^2 and I_1^2 are shown in Fig. 4. The dotted curve represents the distribution in intensity in the focal plane image when there is no aber-

[1] *Camb. Phil. Trans.* **6**, 402, 1838. [2] *Phil. Mag.* (5), **8**, 404, 1879.

TABLE I.

a_0	I_1^2	I^2	a_0	I_1^2	I^2
−2.0	0.0086	0.0000	0.0	0.7093	1.0000
−1.8	.0061	.0108	+0.2	.9158	.8751
−1.6	.0012	.0358	+0.4	.9137	.5754
−1.4	.0004	.0468	+0.6	.6901	.2545
−1.2	.0031	.0243	+0.8	.3629	.0547
−1.0	.0018	.0000	+1.0	.0999	.0000
−0.8	.0017	.0547	+1.2	.0009	.0243
−0.6	.0399	.2545	+1.4	.0395	.0468
−0.4	.1714	.5754	+1.6	.1095	.0358
−0.2	.4159	.8751	+1.8	.1240	.0108
±0.0	.7093	1.0000	+2.0	.0762	.0000

ration (equation 5); the full curve, that when the aberration amounts to ¼ wave-length as assumed above. The effect of the aberration is threefold: (1) it diminishes the intensity at the center of the image; (2) it renders the image asymmetrical with respect to its center of area; and (3) it displaces very sensibly both the position of maximum brightness and of the center of area. The effect of (1) is not serious, but the effect of (2) and more particularly (3) on the accuracy of measurement is most important. The displacement of the point o' (Fig. 4) from the geometrical center o of the undistorted image is about $0.3a$, while the limiting accuracy of measurement, as we have already seen, is about $0.07a$. The displacement o-o' caused by an aberration amounting to only ¼ wave-length is therefore about four times the limit of accuracy ϵ attainable under best conditions.

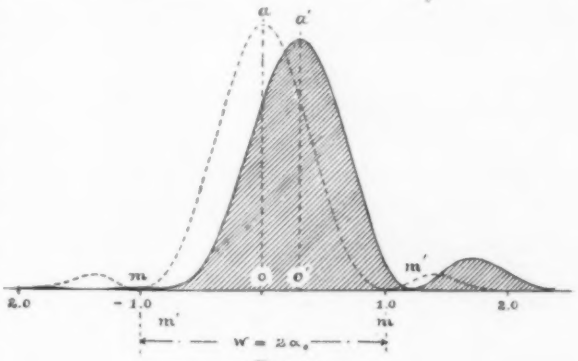


FIG. 4.

An examination of the curve representing a double source, Fig. 5, as viewed under the conditions assumed in Fig. 4, shows that neither the resolving power, which depends primarily on

the form and "width" *mm* of the images of each component, nor the contrasting power, which depends on the width *mm* and the value $o'a'$ of the maximum ordinate, are either of them affected greatly by an aberration of the amount (¼ wave-length) shown. Hence, it is generally assumed that the optical "definition" of an instrument (which, as usually defined, depends principally on resolution and contrast) is not injuriously affected by aberrations which introduce differences of phase in the wave-front of not more than ¼ period, and that therefore optical surfaces which reflect or transmit wave-fronts at nearly perpendicular incidence need to be accurate to ⅛ and ½ wave-lengths only. But as indicated above, this degree of perfection is not sufficient in instruments designed for accurate measurements.

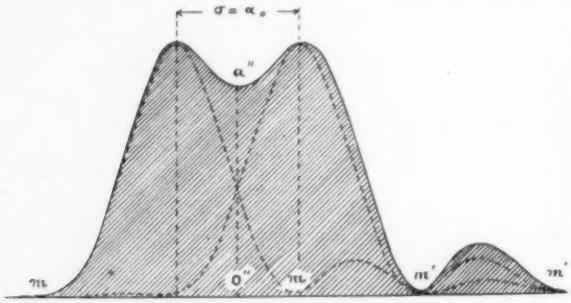


FIG. 5.

The conditions of optical measurement of position are such that we are concerned not so much with the absolute amount of aberration as with the constancy of its effect. For in the great majority of cases we are concerned with the simultaneous or successive determination of the separation or relative position of two images in the same focal field. Under such conditions, if the amount of aberration remained exactly the same for both images, the distortion and displacement of both would also be the same in amount, and in direction, and the measured distance between their centers would be practically unaltered. To avoid errors of measurement, therefore, we need only be certain that the *differential* effect of the wave disturbance on the two images is less in amount than the limit of accuracy ϵ already established. The degree of constancy required is readily determined from the results of the preceding investigation. The displacement of the center of the image in Fig. 4 is, as already stated,

about 0.3a for a phase difference of ¼ wave-length. This displacement is very nearly proportional to the tilting of the actual wave-front with reference to the true spherical wave-front, and this tilting in turn is very nearly proportional to the phase differences between center and edge. Hence, if the differential displacement of the two images is not to exeed the limiting value of ϵ (0.07a), the aberration must not vary by more than

$$\frac{1}{4} \cdot \frac{0.07}{0.30} \cong 0.06\lambda. \qquad (14)$$

The amount of aberration (*i. e.*, the distortion of the wave-front forming the image) will vary at any instant of time: I, by reason of changes in position of an image in the focal field, involving a change in the inclination of the axis of the wave-front to the various optical surfaces and possibly a shift of the wave-front as a whole with reference to those surfaces; II, because of changes in the optical constants of the media traversed by the wave before reaching the focal plane. We will now examine the instrumental and operative conditions necessary to keep the variations due to either cause within the limits given.

I. (1) If the two images are separated by a considerable angular interval in the field of the observing telescope, the two wave-fronts by which they are formed will meet the optical surfaces of the objective at different angles, and the resultant distortion or curvature in each will vary by reason of this fact, even though they were precisely similar before incidence. The amount of aberration introduced in any given case is determined by calculation of the relative retardations of the extreme, with reference to the central rays, which meet at the focal plane.

Thus in Fig. 6 let *abb'* be the points at which the center and extreme edges of the given wave-front x_1x_2 meet the front surface of the optical system, and o', the best-defined center of curvature of the wave-front x_1x_2, after its passage through the optical system. Let *hb* be the distance from the point *b* to the instantaneous wave-front incident at *a*, and *ah* the distance from the instantaneous wave-front incident at *b'*. Then for the pri-

mary plane[1] the retardation of the extreme with reference to the central rays, *i. e.*, the amount of aberration for the two halves of the wave-front, will be represented by

$$\left.\begin{aligned} &ao' - (hb + bo') \\ \text{and} \quad &ao' - (b'o' - h'a) \end{aligned}\right\} \qquad (15)$$

The determination of the value of these quantities in any given case demands a knowledge of the indices of refraction, radii of curvature, and thickness of the lenses forming the optical system, and is usually quite complicated. We can, however, draw certain conclusions from the conditions of the problem which are of more general interest than the detailed consideration of many individual cases.

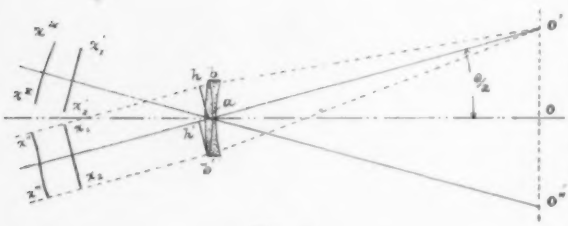


FIG. 6.

The displacement of the centers of two images o' o'' will be in opposite directions if they are on opposite sides of the principal optical axis ao, and will be equal in amount if they are at equal distances from this axis, and both the optical system and the incident wave-fronts, x_1 x_2 $x_1'x_2'$, are symmetrical about their respective axes. Under such conditions of symmetry the distortion of the field itself is symmetrical with respect to its center o, and if these conditions could always be maintained, the proper corrections could be determined once for all and then always applied to the measurements. But the form of the incident wave-fronts is different at different times, as shown at $x''x'''$, $x^{iv}x^{v}$. These changes may be due to influences which are outside of and independent of the instrument itself; and the displacements and the required corrections will therefore vary by an amount which is indeterminate. Our only complete safeguard against errors of this kind is to confine our measurements to a field within which the aberrational distortion and displacement of the lens system itself is negligible. We can form a general idea of

[1] We confine our attention to the aberration at the primary focus because in the case of lines perpendicular to the xy plane the best-defined image is formed at that point.

what this field is in different cases without investigating each one in detail by the following considerations.

The requirements for obtaining the maximum practicable accuracy demand, as has been shown above, that the effect of aberration on the image shall not be more than one-fourth as great as that which would begin to sensibly affect good "optical definition". If we assume that the aberration due to a good lens system varies as the square of the distance from the center, and if we call θ the "field of good definition" (visual or photographic) of the given lens, then the "field of measurement," on the above assumptions, should not exceed $\frac{1}{2}\theta$.

The value of θ depends on the linear size of the objective, b, on its semi-angular aperture, $\beta = \frac{b}{2f}$, and on the type of optical construction. In general we may say that θ varies approximately as $\frac{1}{\sqrt{b}}$ and as $\frac{1}{B}$ for a given type of instrument. The maximum values of θ that have been so far attained with several types of lenses of different apertures are given in Table II.

TABLE II.

Type of Instrument	Size of Objective b	Angular Aperture β	Field θ	
			Visual	Photographic[1]
1. Photographic doublet (new form)[2]	4 in.	1:5 = 0.20		about 15°
2. Photographic doublet (symmetrical construction)[2]	4	1:5 = 0.20		about 8°.5
3. Photographic triplet (symmetrical construction)	2	1:8 = 0.12+	about 2°.5	about 3°
4. Ordinary achromatic (visual)	3	1:12 = 0.083	about 2°.0	
5. Ordinary achromatic (visual)	6	1:18 = 0.055	about 2°.0	
6. Ordinary achromatic (photographic)	15	1:15 = 0.067		about 1°.5
7. Ordinary achromatic (visual)	40	1:18.5 = 0.054	about 0°.7	

[1] The available good field of an objective is always slightly larger photographically than visually on account of the somewhat less rigorous standard of test that can be applied.

[2] See *Annual Reports of the Allegheny Observatory*, 1899, p. 7; 1900, p. 19.

These values of θ are based on experiments and tests made by the writer and others on a large number of objectives of different sizes and types of the best modern construction. They are considerably smaller in some cases than those claimed by the makers of astronomical and photographic objectives, probably because the standard of "good definition" is higher. Many commercial photographic lenses, for example, "cover" fields of 40° or 50° with satisfactory results on an ordinary photographic standard. But when the rigorous tests of stellar photography are applied, the definition of these lenses breaks down only a few degrees from the axis; in fact, I have never found any of them which give results at all comparable with those designed by Hastings, for example, especially for astrophotographic work. A similar statement applies to the longer focus type of objectives (4) and (5). With a low-power eyepiece the visual definition is apparently quite good over fields considerably larger than those given, but with high powers and more careful tests the fields are more restricted.[1]

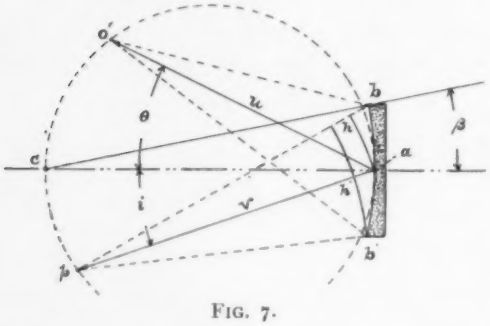


FIG. 7.

The available fields of measurement under the above standards vary therefore from a maximum of about 7° for (1) to about ⅓° for (7).

There is one special case which has recently been examined[2]

[1] The fields given above are those obtainable in photographic use on flat-plates or in visual use without refocusing. These are the conditions that are generally imposed in metrological work. If we use curved plates (or in visual work curved micrometer slides) the field of good definition can be very greatly extended. Thus with a symmetrical doublet of 4½ in. aperture and 20 in. focal length ($\beta = 4.4$) it is possible by the use of a curved plate to cover sharply a field of about 33° diameter, or with a somewhat smaller angular aperture and a special construction of objective, a field of over 40° diameter. See *Annual Reports, Allegheny Observatory;* also "Efficiency of the Curved Plate for Charting Purposes," *Observatory*, **24**, 274 and 419, 1901.

[2] "Aberration of Mirrors and Concave Gratings at the Principal Focus," to be soon published in *Phil. Mag.*

the results of which are of particular interest. This is the case of the reflecting telescope and its associated spectroscopic instrument, the concave grating. The character and amount of aberration here depend on the form of the wave-front. We will consider first the case in which this is spherical as in the ordinary use of the concave grating. The general optical conditions involved in this problem are shown in Fig. 7.

Let p be the position of the radiant point, o', that of its image (either specular or spectral); and c, the center of curvature of the spherical surface[1] bab'. Put $pa = v$; $o'a = u$; $ac = \rho$; $pac = i$; and $cao' = \theta$.

Then from the general theory of optics we have always the relation

$$\frac{\cos^2 i}{v} + \frac{\cos^2 \theta}{u} = \frac{\cos i + \cos \theta}{\rho} . \qquad (16)$$

With Rowland's mounting this relation is satisfied by making

$$u = \rho \cos \theta \quad \text{and} \quad v = \rho \cos i ,$$

and o', c, p, all lie on the circumference of a circle whose diameter is equal to ρ. The aberration at o' for this case has been calculated by Rowland[2] and Glazebrook.[3] The latter finds for the relative retardation for the central ray $(u + v)$ relatively to the edge ray $pb + bo'$,

$$E_1 = pb + bo' - (u + v) = \rho \sin \beta (\sin i - \sin \theta) + \frac{1}{8} \rho \sin^4 \beta (\sin \theta \tan \theta + \sin i \tan i) , \qquad (17)$$

where β is the semi-angular aperture of the surface bab' measured at the center of curvature C; therefore the angle bca. The first term of this expression is that defining the relation between the order of the spectrum m, the wave-length of the spectral line λ, and the number of lines on the grating surface N, *i. e.*,

$$\rho \sin \beta (\sin i - \sin \theta) = \frac{1}{2} Nm\lambda . \qquad (18)$$

The second term is the aberration of the upper half of the

[1] The case of spherical surfaces only is considered here, because that is the form of surface generally used for concave gratings. The case of parabolic and other forms of surfaces is, however, considered at length elsewhere. See paper above referred to.

[2] *Phil. Mag.* (5), 16, 210. [3] *Ibid.*, 377.

spherical surface ab. For the lower half of this surface ab' we find similarly

$$E_2 = (pb' + b'o) - (u + v) = \rho \sin \beta (\sin i - \sin \theta) + \tfrac{1}{8} \rho \sin^4 \beta (\sin \theta \tan \theta + \sin i \tan i) = E_1 . \qquad (19)$$

The aberration is therefore symmetrical, and its effect on the image is simply to broaden it symmetrically without displacing its center.

If the reflection is specular, $\theta = i$. Hence, if we make $u = v$ we always have

$$\rho = u \cos \theta = v \cos i .$$

The first term of (17) and (19) disappears, and the aberration (which is expressed by the second term) becomes

$$\frac{\rho}{4} \sin^4 \beta \sin \theta \tan \theta , \qquad (20)$$

which is likewise always symmetrical.

In case the images, either spectral or specular, are formed on the circle passing through the center of curvature of the mirror as above assumed, we can therefore use any extent of field desired without introducing any error of measurement due to the aberration of the spherical surface itself. The only limit to the available field is that imposed by condition of good definition, *i. e.*, that the aberration shall not exceed a quarter wavelength. This condition applied to (17), (19), and (20) gives

$$\frac{\sin^2 \theta}{\cos \theta} + \frac{\sin^2 i}{\cos i} \leqq \frac{2\lambda}{\rho \sin^4 \beta} . \qquad (21)$$

In the case of Rowland's large gratings $\rho \cong 650$ cm and $\beta = \frac{2\rho}{a} \cong 0.011$. For the maximum value of $i = 60°$ we have therefore

$$\frac{\sin^2 \theta}{\cos \theta} \cong 9.5 \quad \text{or} \quad \theta = 84° 15' .$$

For gratings and mirrors of large angular aperture, such as are sometimes used in stellar spectroscopic work, the maximum value of both θ and i are considerably reduced. Thus for a

grating of 150 cm radius, of curvature and linear aperture the same as before, $\beta = 0.050$. We have then

$$\frac{\sin^2\theta}{\cos\theta} + \frac{\sin^2 i}{\cos i} = 0.106 \ .$$

which shows at once that neither θ nor i can in this case exceed $18°20'$. If we use the Rowland mounting for which $\theta = 0$ for the center of the field and make $i = 15°$ we have in the above case

$$\theta_{max} \cong 11° \ .$$

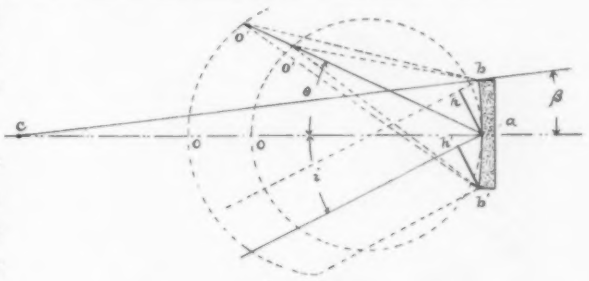


FIG. 8.

For the concave grating the maximum field is of course obtained by making $i = 0$, and placing the photographic plate or eyepiece directly above the slit. Under such circumstances we can obtain good definition over a field 36° long (18° on each side of the center).

In the case of the reflecting mirror the maximum field is determined by making $\theta = i$ in (21). For a mirror of the dimensions last considered we have therefore

$$\theta_{max} \cong 12° 30' \ ,$$

or about ⅔ as large as obtainable with the concave grating.

Second case—incident wave-front plane. This case corresponds to the ordinary use of the reflecting telescope, and to the use of the grating as suggested by the writer in 1896.[1]

The optical conditions of this problem are shown in Fig. 8, the notation being the same as before.

The relative retardation of the central with reference to the extreme rays is in this case

$$\begin{aligned} E_1 &= u - (bo' + bh) \text{ for the upper half of the surface,} \\ E_2 &= u - (bo' - ah') \text{ for the lower half of the surface.} \end{aligned} \qquad (22)$$

[1] ASTROPHYSICAL JOURNAL, **3**, 55–60. Since this article was written the concave grating has been used in the manner suggested, by Poor and Mitchell (*ibid.*, **7**, 157; **10**, 29), in stellar spectroscopic work, and by Jewell; Mohler & Daniel, (*ibid.*, **12**, 361); Frost (*ibid.*, **12**, 311), and by the writer (*Report Allegheny Observatory*, 1900, p. 23), in solar-eclipse work.

The general expressions for E_1, E_2 can be derived from the geometry of the triangles bao', $ab'o'$, bah, and $a'b'h'$. Using the same notation as before we find as general expressions for E_1 and E_2

$$\begin{aligned}
E_1 &= \rho \sin\beta (\sin\theta - \sin i) \\
&+ \tfrac{1}{2}\rho \sin^2\beta \left\{ \cos\theta + \cos i - \frac{\rho}{u}\cos^2\theta) \right\} && = B \\
&+ \tfrac{1}{2}\frac{\rho^2}{u^2}\sin^3\beta \left\{ \sin\theta\cos\theta\,(u - \rho\cos\theta) \right\} && = C \\
&+ \tfrac{1}{2}\frac{\rho^2}{u^2}\sin^4\beta \left\{ \begin{array}{l} \rho\sin^2\theta\cos\theta - \dfrac{\rho}{u}\sin^2\theta\cos^2\theta - \dfrac{1}{4}u \\ + \dfrac{1}{4}u\cos^2\theta + \dfrac{1}{4}\dfrac{\rho^2}{u}\cos^4\theta - \dfrac{1}{2}\rho\cos\theta \end{array} \right\} && = D \\
&+ \tfrac{1}{8}\rho\sin^4\beta\,(\cos\theta + \cos i) && = F
\end{aligned} \qquad (23)$$

and

$$E_2 = \rho\sin\beta(\sin i - \sin\theta) + B - C + D + F\,. \qquad (24)$$

In this case the aberration is unsymmetrical, the amount of asymmetry being expressed by the term in $\sin^3\beta\,(C)$. The value of this term depends on the manner in which the grating is used. Three cases may be distinguished, as has been already indicated:

(a) When the direction of reflection is parallel to the line of incidence pa.[1] Then $\theta = -i$, and from (16) we find for u, $(v = \infty)$,

$$u_a = \frac{\rho}{2}\cos\theta\,. \qquad (25)$$

In this case we have for the term C

$$\rho\sin^3\beta\sin\theta\,. \qquad (26)$$

(b) When we examine the spectra in the neighborhood of the center of curvature. $\theta = 0$ for the center of the field, and we have for u [2]

$$u_b = \frac{\rho}{1 + \cos i}\,. \qquad (27)$$

Then for (C) we find for small values of θ

$$\rho\sin^3\beta\sin\theta\,\frac{1}{2}\cos i\,(1 + \cos i)\,. \qquad (28)$$

[1] Astrophysical Journal, 3, 58, 59. [2] *Ibid.*, pp. 54–57.

(*c*) When we make the incidence on the grating normal. Then $i = 0$ and

$$u_c = \frac{\rho \cos^2 \theta}{1 + \cos \theta} , \tag{29}$$

and for (*C*)

$$\rho \sin^3 \beta \sin \theta \frac{1}{2} \left(\frac{1 + \cos \theta}{\cos^2 \theta} \right) . \tag{30}$$

(*d*) Finally for regular specular reflection $\theta = i$, and

$$u = \frac{\rho}{2} \cos \theta , \tag{30}$$

just as in case (*a*).

As the aberration is in all these cases unsymmetrical, the available measurable field is found by equating (14) with (26), (28), and (30) (neglecting terms in $\sin^4 \beta$).

This gives us for cases (*a*), (*b*), and (*d*)

$$\sin \theta_a = \frac{\lambda}{16 \rho \sin^3 \beta} K . \tag{31}$$

In the case of Rowland's gratings, already discussed, for which $\rho = 650$ cm. and $\beta = 0.011$, the value of θ given by (31) is

$$\theta_{\max} \cong 2^\circ 20' ,$$

when the grating is used as in (*a*). When it is used as in (*b*), and we make the maximum angle of incidence $i = 60^\circ$, we have for θ.

$$\theta_{\max} = \theta \frac{2}{\cos i \, (1 + \cos i)} \cong 6^\circ .$$

Finally, if we use the same grating as in (*c*), we find for θ

$$\theta_{\max} = \theta_a \frac{2 \cos^2 \theta_a}{1 + \cos \theta_a} \cong 2^\circ 20' \text{ as before} .$$

Another case of great importance and interest is the use of mirrors as collimators or telescopes of spectroscopes. If the mirrors are of long focus, very satisfactory results are attained by their use. Thus in one of the instruments used by the writer the spherical mirror had an aperture of 6 cm, and a focal length of about 175 cm. For such a mirror we have $\rho = 350$, $\beta \cong \frac{1}{117}$.

With these values we get from (26)

$$\sin i = 0.016 \quad \text{or} \quad i = 55' .$$

The diameter of undistorted field is therefore about 1°.8. The diameter of the field of good definition (for qualitative work only) is of course about four times this, or about 7°.5. These results agree well with those actually found in practice,[1]

For large angular apertures the results are much less satisfactory. Thus for a surface such as was considered before for which $\rho = 150$ cm and $\beta = 0.05$ we have

$$\theta_a = \theta_d \cong \theta_c \cong 40'' .$$

By mounting the grating as in (b) and using a very large value of i ($i = 75°$) we may increase this field about six times, *i. e.*,

$$\theta_b \cong 2' .$$

We may likewise increase the field by diminishing the focal length (keeping β the same) in the proportion $\frac{1}{\rho}$. For a grating of only 1 inch aperture and 5 inches focal length $\beta = 0.05$ as before and $\rho = 12.5$cm. The value of θ_b for this case is

$$\theta_b \cong 25' .$$

The field of good definition in all the above cases is of course about four times as large as the field of negligible measurable distortions.

The above results show clearly that the use of spherical gratings or mirrors of very large angular aperture with parallel incident light is impracticable for either quantitative or qualitative work, except in the case of very small gratings used as in (b). If such mirrors or gratings are to be used of large size, it is absolutely necessary that their surfaces be specially figured for the particular use desired. This question is considered at greater length elsewhere.

I. (2) If the wave-fronts which form the two images are appreciably shifted in lateral position so as to fall upon different portions of the optical train, it is at once evident from (14) that the mechanical errors in the different surfaces involved must not exceed the amount that would introduce a difference of phase of 0.06λ. If t be the permissible mechanical error and ϕ the angle of incidence of the wave-front, we have at once

[1] "An Improved Form of Littrow Spectroscope," *Phil. Mag.* (5), 38, 137–142, 1894.

$$\left.\begin{aligned} t &= \frac{0.06\lambda}{2\cos i} \quad \text{for reflecting surfaces} \\ t &= \frac{0.6\lambda}{n\cos i' - \cos i} \quad \text{for refracting surfaces} \end{aligned}\right\}^{1} \tag{32}$$

Two cases of importance that need to be considered in this connection are those of the heliometer and the spectrograph. In the first instrument the two images which are brought to coincidence are formed by entirely different portions of the objective, and the optical surfaces of each portion must therefore be correct to the limit indicated in (32). Since i and i' are both nearly 90°, the two surfaces of the flint lens must each be optically right to within about $\frac{1}{10}\lambda$ and the two surfaces of the crown to within about $\frac{1}{8}\lambda$ each. The standard of workmanship required in a heliometer objective is therefore very high, and the difficulty of attaining the necessary degree of perfection is rendered vastly more difficult by the division of the finished object-glass into two parts. If there is the slightest strain or difference in density in the material, the glass is likely to spring slightly along the line of cutting, and this line is unfortunately parallel to the axis of measurement.[2]

In the case of the prism spectroscope and spectrograph the wave-fronts which form the images of different spectral lines are shifted laterally with reference to each other in passing through the train by the amount

$$\sum_{o}^{L} l\frac{d\theta}{d\lambda} = S\ , \tag{33}$$

where l is the length of path traversed by the wave-front between each successive refraction, beginning with the incident side of the first prism and ending with the camera or view telescope objective. For a prism train consisting of N similar

[1] RAYLEIGH, "On the Accuracy Required in Optical Surfaces," *Phil. Mag.* (5), **8**, 477.

[2] The form of prism heliometer which has recently been devised and constructed by the writer avoids this difficulty by rendering the division of the object-glass unnecessary. It can therefore be constructed of much larger size than has been possible with the older form. See *Allegheny Observatory Report* for 1900, p. 23; also U. S. Patents, Nos. 536493, 536494, 536555.

prisms with their corners in contact, we have for the shift from the central ray which passes at minimum deviation

$$\left.\begin{aligned} S = \frac{1}{2}\frac{d\theta}{d\lambda}\Big[& B_1 + 3B_2 + 5B_3 + \ldots + (2N-1)\,B_N \\ & + 2nB_1 + 4nB_2 + 6nB_3 + \ldots + 2Nn\frac{B_N}{2} \end{aligned}\right\}, \qquad (34)$$

where B_1B_2, etc., are the lengths of the bases of the prisms. For some of the larger instruments recently constructed the value of S is quite large. For the Bruce spectrograph of the Yerkes Observatory, for example,

$$N = 3\,;\ n \cong 1.67\,;\ B_1 = 12\,\text{cm}\,;\ B_2 = 13\,\text{cm}\,;\ B_3 = 14\,\text{cm}\ .$$

The angular separation of the extreme images measured on the plate is about 4°; therefore $\frac{d\theta}{d\lambda}$ (one refraction) $\cong \frac{2}{3}° \cong 0.012$. We obtain therefore for S

$$S_{\max} \cong 2\,\text{cm}\ .$$

The refracting angles of the prisms of this train are about 63° 30′, and μ for the central ray is about 1.673. Hence i and i' are about 61° 45′ and 31° 45′ respectively. For these values we have for t, from (32)

$$t_{\max} = \frac{0.06\lambda}{0.94} \cong 0.07\lambda\ ,$$

or the prism surfaces must be correct to within about $\frac{1}{13}$ of a wave over all those portions not common to the two beams, *i. e.*,

$$2\,\text{cm} \div \cos 61° 45' \cong 4\,\text{cm}\ ,$$

at each end of the prism faces. This would mean that the prism faces should not depart from a perfect plane by more than 0.000003 cm in a length of 6.5 cm, or that the radius of curvature of the faces should be at least 7,000,000 cm. Such a degree of accuracy is very difficult to attain in surfaces whose length (13 cm) is so much in excess of their width (5.1 cm) as in this case, but it is even more difficult to maintain when once secured, if temperature conditions vary rapidly.

In case of the plane grating spectroscope the same portion of the grating surface may always be used, but the angle of incidence and reflection varies with the wave-length. As ordinarily

used with fixed collimator and observing telescope, the angles θ and i are connected by the relation

$$i + \theta = \gamma \ ,$$

where γ is the fixed angle between the axes of the two telescopes, and the angles θ and i are measured positively on opposite sides of the normal as before. In this case any irregularity of surface of height t introduces a retardation which is evidently

$$t(\cos i + \cos \theta) \ . \tag{35}$$

For setting on the central slit image we must have $\cos i = \cos \theta = \cos \frac{1}{2}\gamma$, and the error of phase is

$$2t \cos \frac{1}{2}\gamma \ . \tag{36}$$

For setting on any spectral line of wave-length λ we must revolve the grating through an angle δ such that

$$\sin \delta = \frac{Nm\lambda}{2a \cos \frac{\gamma}{2}} . \tag{37}$$

Also $\theta = \frac{\gamma}{2} - \delta$ and $i = \delta + \frac{\gamma}{2}$, and (35) reduces at once to the form

$$E = 2t \cos \delta \cos \frac{\gamma}{2} \ . \tag{38}$$

In order that the spectral image may not be displaced a measurable amount from its true position with reference to the central slit image, the condition is

$$2t \cos \frac{\gamma}{2}(1 - \cos \delta) \leqq 0.06\lambda \ . \tag{39}$$

If the angle between the two telescopes is 30°, $\cos \frac{\gamma}{2} = 0.966$, and for the image of the D lines in the second order spectrum of a grating with 5,000 lines to 1 cm, the value of δ is

$$\delta \cong 17^\circ 45' ,$$

and for the fourth order spectrum

$$\delta \cong 37^\circ 40' .$$

And from (39) we obtain at once for t

$$t_{\max} \leqq \begin{cases} 0.6\lambda \text{ for the first case,} \\ 0.15\lambda \text{ for the second case,} \end{cases}$$

from which it appears that the irregularities in the surface of a plane grating large enough to introduce a measurable displacement of the image of a spectral line would probably manifest themselves by a deterioration of the optical definition.

II. The effect of temperature variations in the media traversed by the wave-fronts is more serious than those of irregularities of surface or varying obliquity which we have just considered. For the latter would be constant in amount and direction, provided the incident wave-fronts were constant in form, and under such circumstances could be corrected for. But the effect of temperature changes in the various optical parts of the instrument is to change continually the form of the passing wave-front, and thus not only render the corrections in question varying and uncertain, as has already been stated, but also introduce errors of displacement which vary with time alone. This latter class of errors can be eliminated only in two ways: first, by making the measurements on the two images absolutely simultaneous in time; or, second, by keeping the temperature variations within the limits which will render the resulting displacements less than the quantity ϵ. The first condition can be satisfied in certain cases by heliometric methods or by simultaneous photography of the two images, the measurements being afterward made on the photographic plate instead of directly in the focal plane. The elimination of the effect of what may be termed time aberrational displacements in this way is one of the advantages of the heliometric and photographic methods of measurement that does not seem to have been heretofore specifically recognized.

If the above time condition cannot be strictly fulfilled, then the temperature variation must not exceed an amount that would introduce an unsymmetrical aberration of more than 0.06λ (see (14)). We will next examine briefly what this condition of steadiness implies. The change in the optical density (index of refraction) of air and glass for 1° C. is

$$\left.\begin{aligned} \Delta n_a &= -1.1 \times 10^{-6} \text{ for air,} \\ \Delta n_g &= +4.0 \times 10^{-6} \text{ for flint glass.}^1 \end{aligned}\right\} \qquad (40)$$

[1] J. O. REED, "Einfluss der Temperatur auf die Brechung . . . . einiger Gläser," Inaugural Dissertation, Jena, 1897. Some varieties of glass examined by Dr. Reed,

The total difference in phase $\frac{\Delta\lambda}{\lambda}$ introduced in the wave-front in traversing a thickness L raised to a difference of temperature T will evidently be

$$\frac{\Delta\lambda}{\lambda} = \frac{L}{\lambda} T\Delta n \ . \tag{41}$$

Under the condition that $\frac{\Delta\lambda}{\lambda}$ shall not exceed 0.06 we have at once

$$\left.\begin{aligned} LT &\leqq \frac{0.06\lambda}{1.1 \times 10^{-6}} \cong 2.5 \text{ for air,} \\ &\leqq \frac{0.06\lambda}{4 \times 10^{-6}} \cong 0.7 \text{ for flint glass.} \end{aligned}\right\} \tag{42}$$

The focal length L_a of the largest heliometers now in use is about 250 cm, and the average thickness L_g of their objectives (aperture 16 cm) is about 1.5 cm. From (42) we obtain at once

$$\left.\begin{aligned} T_a &\leqq 0^\circ.01 \text{ C.} \\ T_g &\leqq 0^\circ.5 \text{ C.} \end{aligned}\right\} \tag{43}$$

That is, in order to avoid an error of displacement greater than the limit of accuracy $\epsilon = 0.07a$ attainable with this instrument, the temperature of the two halves of the object-glass must remain constant to within $\frac{1}{2}^\circ$ C. and that of the air in the two sides of the tube constant within $0^\circ.01$ C., in the interval between two successive sets of measurements.

In the case of the Bruce spectrograph considered above, L_g (for the prism train) is about 39 cm and the corresponding value of L_a $(39n)$ is about 65 cm. If the temperature of either the air or glass changes alone, the limit of this temperature change must be

$$\left.\begin{aligned} T_a &\leqq 0^\circ.038 \text{ C.} \\ T_g &\leqq 0^\circ.018 \text{ C.} \end{aligned}\right\} \tag{44}$$

If the two change together and by the same amount the limit of the change must be (40) and (41)

$$T \leqq \frac{0.06\lambda}{L_g\Delta n_g - L_a\Delta n_a} = 0^\circ.012 \text{ C.} \tag{45}$$

notably the barium silicate crowns and the light barium flints, have much smaller temperature coefficients and would on that account be excellent for use in this connection.

For the camera objectives, focal lengths $L_a = 45$ cm and 60 cm the limiting values of T are

$$T_a \leqq 0^{\circ}.055\,\text{C.} \quad \text{and} \quad 0^{\circ}.04\,\text{C.} \tag{46}$$

The interval between measurements, which in this case is the interval between the star exposures and the exposure for the comparison spectrum, is so long and the temperature requirements so severe in the case of an instrument as large as the one considered that the only way of avoiding error would seem to lie in first keeping the temperature as constant as possible, perhaps within $0^{\circ}.1$, and exposing *continuously and simultaneously on both the star and the comparison source*. The method now generally used of making one exposure on the latter just before and another just after the exposure on the star is not the equivalent of a continuous exposure on both, because the temperature will not change uniformly in the glass, even if it does in the air, and the effect of an unsymmetrical or irregular change in the former is, as shown above, about twice as great as in the latter.

Case A (3). Effect of variations in the value of λ (non-achromatism of image).

When the value of λ in the general equation (3) varies, we have to take into account not only the resulting change in the *scale* of the diffraction pattern for each wave-length, but also several other effects as follows:

(*a*) The chromatic dispersion of the instrument and the relative inclination (if any) of the different incident wave-fronts corresponding to different values of λ.

(*b*) The spectral distribution in intensity in the different wave-fronts.

(*c*) The relative chromatic sensitiveness of the eye or photographic plate by means of which the position of the image is determined.

(*a*) The effect of chromatic dispersion of the instrument is to alter the inclination of the various wave-fronts passing through it by an amount depending on the wave-length λ and the optical form and constants of the media traversed.

Let us denote this varying inclination of the different wave-fronts by ϕ. Then in general

$$\left.\begin{aligned} \phi &= \frac{d\theta}{d\lambda}\,(\lambda - \lambda_0) = D\Delta\lambda \\ \text{and} \qquad d\phi &= D'\,d\lambda\ , \end{aligned}\right\} \tag{47}$$

where λ_0 denotes the wave-length of the wave-front which is brought to focus on the axis of the geometrical image.

(*b*) The intensity of light in each wave-front will depend on the spectral distribution in intensity in the source of radiation itself, which we may denote by $\psi(\lambda)$, and on the coefficient of transmission for each wave-length, which we will denote by k_λ. The relative intensities in the different wave-fronts will therefore be proportional to

$$k_\lambda \psi(\lambda)\ . \tag{48}$$

(*c*) Finally, the relative effect of the image formed by each wave-front at the focal plane will depend on the "luminosity curve," either visual or actinic, which we may denote by $L(\lambda)$.

Taking into account all of these factors we have for the distribution in intensity in the spectral diffraction pattern of a point or line at the focal plane of an achromatic telescope of rectangular aperture the expression

$$I_s^2 = \int_{-\infty}^{+\infty} k_\lambda \psi(\lambda) L(\lambda) (I_r^2)\, d\lambda\ , \tag{49}$$

where

$$I_r^2 = A \frac{\sin^2 \frac{\pi}{a_0}(a - \phi)}{\left[\frac{\pi}{a_0}(a - \phi)\right]^2} \cdot \frac{d^2}{f^2} \frac{\sin^2 \frac{\pi b}{\lambda}(a - D\Delta\lambda)}{\pi^2 (a - D\Delta\lambda)}\ . \tag{50}$$

from (5).[1]

In the evaluating of the integral I_s^2 of (49) and (50) we have to distinguish several cases:

1. When there is no chromatic dispersion, $D = 0$, and if the different wave-fronts all fall concentrically on the diffracting aperture, the centers of the different spectral images will coin-

[1] See "General Theory of Telescopic Images for different forms of Radiating Sources," ASTROPHYSICAL JOURNAL, **6**, 124; § § A (*b*) and A (*c*).

cide, or more strictly, will all fall on the optical axis passing through the center of the geometrical image. Since each individual image is a symmetrical function of λ, the diffraction pattern resulting from their centrical superposition will also be symmetrical, and the result of the residual chromatic effects, (*b*) and (*c*), as well as any outstanding longitudinal chromatic aberration will be to change the *form*,[1] but not the position of the center of intensity of the physical image. This effect, like that of A (1), will not therefore affect the accuracy of measurement of positions.

2. When the chromatic dispersion is small the superposition of the different spectral images will not be quite exact, but the continuity of the image will be unbroken; *i. e.*, the chromatic resolution will not be sufficient to enable us to isolate or deal with any individual line or region of the spectrum. Under such circumstances there is considerable uncertainty as to just what part of the image corresponds in position to the geometrical center, and the error of measurement involved in the determination of the position of the source itself may be very appreciable. Such a case as is here supposed presents itself in micrometric and heliometric measurements of the relative positions of stars and small planets at low altitudes. In every position except exactly in the zenith (and even there under certain conditions of barometric pressure) the Earth's atmosphere acts as a prism, which not only results in the refraction, but also in the dispersion of the light coming from the star or planet. The necessity of considering the dispersion effect was early pointed out by Lee,[2] and was afterwards more fully discussed by Rambaut[3] and Gill[4], who do not, however, fully agree as to the magnitude of the effect involved. Their disagreement can be explained in

[1] As has already been indicated this change in *form* may fortunately be utilized as the basis of a possible method of measuring stellar temperatures. *Loc. cit.*, § A, p. 125.

[2] *Phil. Trans.*, 1815.

[3] "Effect of Atmospheric Dispersion on the Position of a Star." *M. N.*, **45**, 123–145; *ibid.*, **48**, 256–280.

[4] "Effect of Chromatic Dispersion of the Atmosphere on Parallaxes." *Ibid.*, **48**, 53–76; *ibid.*, 415–425.

part by their failure to consider fully the effect of the factors k_λ, $\psi(\lambda)$ and $L(\lambda)$ of (*b*) and (*c*) above, which are quite as important in determining the apparent center of the image of the star as is the term $\phi = D\Delta\lambda$ which represents the atmospheric dispersion. Thus Rambaut shows that certain systematic differences in the measurement of the angular separation of the components of *β Cygni*, made at different zenith distances, may be explained if we take into account the factor $D\Delta\lambda = D(\lambda_o - \lambda)$ only, by supposing that the value of λ_o for the two stars (or, as he puts it, the mean refrangibility) differs by about 250 tenth-meters. Differences of this amount are easily possible between stars of such different types as the two components of the star in question, but the change in apparent separation of their images due to this difference in mean wave-length of maximum intensity is considerably less than Dr. Rambaut assumes, owing to the influence of the omitted factors k_λ, $\psi(\lambda)$, and $L(\lambda)$, in determining the apparent center of the image when the latter is observed either visually or photographically. This is very fortunate, since otherwise, as Dr. Gill points out, observations of parallactic displacement would be affected by errors or rather uncertain corrections of so great a magnitude as seriously to reduce the accuracy of such work. At the same time there remains a true apparent displacement of the center of intensity of the image which is, in many cases, greater than the limiting error of measurement ϵ. Whether this displacement of the *center of intensity* affects heliometer measurements to the same extent that it affects micrometer observations is, as Dr. Gill points out, a question that can be settled only by individual experiment.

The whole question is of vital interest not only in connection with parallax and double-star work, but also in connection with meridian work and almucantur observations and (under certain conditions as already indicated) zenith telescope measurements. I have therefore evaluated the integral (49) for a number of values of λ_o and under a number of assumptions as to the form of the functions k_λ, $\psi(\lambda)$, and $L(\lambda)$ for both visual and photographic telescopes. This work would occupy too great a portion of the present paper and I will therefore present it in a separate communication.

3. When $\frac{d\theta}{d\lambda}$ is large, as in the case of the spectroscope, we no longer have to deal in general with the whole spectral image of the source, but only with individual portions of it, *i. e.*, with individual spectral lines. In practically all cases the effective "spectral width" $\Delta\lambda$ of individual lines is so narrow that over this range the dispersion coefficient $D=\frac{d\theta}{d\lambda}$ may be regarded as constant, and for the same reason the value of λ itself may be considered as constant with respect to the integration of (49) in the functions k_λ, $L(\lambda)$ and in the denominator of (50). Under such circumstances the expression for the distribution in the intensity in the image of an individual spectral line becomes

$$I_1^2=A'\int_{-\infty}^{+\infty}\psi(\phi)\frac{\sin^2\frac{\pi}{a_0}(a-\phi)}{\left[\frac{\pi}{a_0}(a-\phi)\right]^2}d\phi=A''\psi_1(\psi,a_0,a)\ , \qquad (51)$$

or if the source (in this case the slit of the spectroscope) has a finite width[1] σ_0,

$$I_2^2=A''\int_{-\frac{\sigma_0}{2}}^{+\frac{\sigma_0}{2}}\psi_1\{\xi-\psi,a_0,a,\}\,d\xi\ . \qquad (52)$$

In (51) and (52) $\psi(\phi)=\psi(\lambda)$ represents the spectral law of radiation not for the source as a whole, but for any individual element. In a normal source whose mean wave-length is λ_0 the distribution $\psi(\lambda)$ is generally assumed to be represented by the law

$$\psi(\lambda)=e^{-k(\lambda_0-\lambda)^2}\ , \qquad (53)$$

which is symmetrical in form. For such sources I_1 is symmetrical and I_2 is therefore also symmetrical. This case has already been considered.[2]

When $\psi(\lambda)$ is unsymmetrical, as is generally the case when the source of radiation is subjected to the effect of unusual pressure or temperature conditions, or to abnormal magnetic or electrical disturbances, the form of I_2^2 and I_2^2 likewise become

[1] See *Phil. Mag.*, **43**, 330, 338; also ASTROPHYSICAL JOURNAL, **3**, 336.

[2] *Loc. cit.*, p. 330ff.

asymmetrical. Unfortunately our knowledge of the relations between the physical conditions of molecular and atomic vibrations and the resultant intensity of radiation are as yet too meager and unsatisfactory to enable us to express $\psi(\lambda)$ in definite mathematical form in such cases. We can only consider and endeavor to allow for the general effect produced, and this will be to render the measurement of the mean wave-length, λ_o, of a given line indeterminate, to a degree depending on physiological and psychological causes rather than on physical ones. That is, different observers will differ among themselves as to the setting of a cross-wire on the mean center of intensity of such an asymmetrically broadened image, but aside from this, this kind of asymmetry cannot be said to be a real source of error in the measurement of wave-lengths, since the latter can only be considered in reference to the line itself, and not, at least in the present state of our knowledge, in reference to the free or natural period of the vibrations which have produced it[1].

[1] In this connection see papers by JAUMANN, "Zur Kenntniss des Ablaufes der Lichtemission." *Wied. Ann.*, **53**, 832; **54**, 178; and by GALITZIN, "Zur Theorie der Verbreiterung der Spectrallinien," *ibid.*, **56**, 78. Further developments of the theory of radiation along the lines outlined in these papers seem probable.

(*To be concluded.*)

RESEARCHES ON THE ARC SPECTRA OF THE METALS.

VI. SPECTRUM OF MOLYBDENUM.[1]

By B. HASSELBERG.

INTRODUCTION.

THE comprehensive classical investigation of the solar spectrum by Rowland has not only furnished to spectroscopic research of our times a foundation of a relative accuracy hitherto unattained and to all appearances hardly to be surpassed for a long time, but has also by the simultaneous study of the spectra of the metals brought the question of the chemical constitution of the Sun very much nearer to a solution. Although our knowledge as to the source of the Fraunhofer lines has been decidedly increased, Rowland's catalogue of wave-lengths nevertheless itself gives the best testimony as to how much remains for research in this field before the chemistry of the Sun, and consequently that of the stars, can be brought to even a provisional conclusion. Of the seventy odd chemical elements, only about a half are represented in the absorbing envelope of the Sun, while a majority of the remainder have not permitted any identification, and a few in only a doubtful manner. There is, however, no reason to regard this condition of things as a permanent one, for continued researches in the field of pure spectroscopy will doubtless lead to a considerable extension of what has already been attained, particularly as Rowland's investigations of the spectra of the metals are certainly not to be regarded as equal in completeness to his researches on the solar spectrum, and further, the spectroscopic peculiarities of entire groups of metals, particularly the so-called rare elements, have been hitherto practically wholly unknown.

In this category of spectroscopically neglected metals belong the three elements, molybdenum, tungsten and uranium, which

[1] *Kongl. Svenska Vetenskaps-Akademiens Handlingar*, 36, No. 2, 1902.

on the basis of their chemical properties are included in the iron group. Rowland records the first of these as certainly present in the solar atmosphere, but the other two as doubtful. This statement, to which I shall recur below, is the more remarkable since the remaining members of the iron group—iron, nickel, cobalt, chromium, and manganese—are just the metals that appear to be the most numerously represented in the general solar spectrum. In order to obtain more definite information as to these conditions, which, moreover, also occur in the case of other groups of metals, and in order to complete my former investigations on the spectroscopy of the iron metals, I have considered a thorough investigation of the arc spectra of the above-named metals as particularly desirable, and this especially since these spectra have hitherto remained entirely unknown, aside from the small region between λ 3900 and λ 4000 studied by Lockyer.[1] The fact that the corresponding spark spectra have been investigated, first in general outline thirty years ago by Thalén,[2] and then for the violet and ultra-violet portions in a very commendable manner by Exner and Haschek,[3] and for uranium recently by Lohse,[4] can in my opinion only lend an increased interest to such an undertaking. Now that these investigations have been concluded within the limits prescribed by my apparatus and also by the present needs of astrophysics, for molebdenum and have been carried out to a considerable extent for tungsten, it seems a proper time to communicate the results obtained, and I therefore beg to lay before spectroscopists in the following pages the results for the first-named metal.

In my previous papers I have fully described the instrumental equipment employed, the method of obtaining and measuring the photographic plates as well as of eliminating foreign lines, and, as no important changes have been made in these respects, I may therefore omit further discussion of this matter. In reply to a suspicion recently expressed[5] that the grating I have used

[1] WATTS' *Index of Spectra*, 1889. [2] *Nova Acta Reg. Soc. Scient.* Upsala, 1868.

[3] *Wien. Sitz.-Ber.*, Mathem. naturwiss. Classe, **104**, 1895; **105**, 1896; **106**, 1897; **107**, 1898.

[4] *Berlin. Sitz.-Ber.*, 1897.

[5] *Wien. Sitz.-Ber.*, Mathem.-naturw. Classe, **110**, 986, 1901.

had a very slight dispersion, I would remark here that this grating, which Professor Rowland at the time had the kindness to select for me as the best among a considerable number, has a ruled surface of 8×5 sq. cm, with 14,438 lines to the inch, and that it constitutes in connection with an excellent Steinheil telescope of 85 mm aperture and 1.6 m focus as collimator and with a camera having an objective of the same dimensions, a spectrograph which in respect to its optical power is only surpassed by the large Rowland concave grating. For this reason I have hitherto employed this apparatus exclusively for my investigations of the arc spectra; but I have also recently mounted a large concave grating, of 6.3 m radius and 20,000 lines to the inch, in order to secure the necessary efficiency in the extreme ultra-violet, which cannot be reached by the present spectrograph on account of the absorption in the glass. Judging by the provisional tests of this grating, I have every reason to expect the best results from its use.

ELIMINATION OF FOREIGN LINES.

In my first attempt to obtain the arc spectrum of molybdenum I introduced fragments of molybdenum sulphide into the arc, at the suggestion of Baron Nordenskiöld, in the hope of thus obtaining the purest possible spectrum of molybdenum, since the mineral in question was said to contain in addition to molybdenum only sulphur, which is known to yield no spectrum in the arc. But my expectation was not confirmed, for a superficial examination of the plates exhibited so serious a contamination by foreign lines that the further use of this material had to be given up. I substituted for it pure metallic molybdenum from the chemical works of Merck in Darmstadt, and thus obtained a spectrum of decidedly greater purity, although in this case, as in all similar ones, chemistry is still far from being able to supply a spectroscopically pure preparation. The number of foreign lines therefore still remains not inconsiderable, in spite of the pains taken to suppress them in a spectrum having so many lines as the present one, but I nevertheless hope that most of the impurities have been removed from the catalogue of lines to be

given below. The comparisons made with previously known arc spectra, for accomplishing this elimination, give the following results:

MOLYBDENUM AND IRON.

On the first examination of the plates the majority of the easily recognizable iron lines were excluded from the provisional list of wave-lengths for molybdenum. The comparison with the iron spectrum of Kayser and Runge after the completion of the definitive catalogue yielded a considerable number of instances of approximate coincidence, the reality or unreality of which were tested, line for line, by the accurate investigation of plates exposed to the two spectra especially for this purpose. The results of this investigation are shown in the following table:

Mo λ	*i*	*Fe* λ	*i*	Remarks
3466.98	2	3467.05	5	Separated. λ *Mo* < λ *Fe*
81.95	1.2	81.94	6	*Fe* line lacking
3493.49	2−	3493.44	6	Separated. λ *Mo* > λ *Fe*
3504.55	2.3	3504.56	6	*Fe* line lacking.
24.35	2+	24.38	5	Separated. λ *Mo* < λ *Fe*
26.08	1+	26.12	5	Separated
52.57	1	52.62	6	Probably λ *Mo* < λ *Fe*. Rowland has 52.57, .69
54.35	2−	54.28	4	Separated. λ *Mo* > λ *Fe*
66.57	1	66.50	5	Separated
71.42	1.2	71.38	5	Separated. λ *Mo* > λ *Fe*
74.05	2.3	74.04	3	Coinc. Other *Fe* lines in the vicinity are lacking in *Mo*
82.03	3.4	81.98	6	Separated. *Mo* line dupl.
90.90	2+	90.84	6	Coinc. or perhaps λ *Mo* <λ *Fe*
91.55	1.2	91.52	6	Separated. λ *Mo* > λ *Fe*. Rowland has 91.50, .63
94.73	1	94.75	3	Separated. λ *Mo* < λ *Fe*
3595.87	2	3395.82	6	*Fe* line lacking
3603.86	1.2	3603.84	6	
10.80	1+	10.87	3	Separated. *Mo* line exceedingly faint
14.87	1	14.79	6	Coinc.? *Fe* line diffuse
23.36	2+	23.34	3	Separated. λ *Mo* >λ *Fe*. Rowland gives 23.36
26.33	2.3	26.31	6	Coinc.
55.21	1.2	55.13	6	Separated. λ *Mo* > λ *Fe*
63.14	2	63.05	6	*Fe* line lacking
77.83	2−	77.78	4	No coinc. λ *Mo* > λ *Fe*
81.88	2−	81.80	6	*Fe* line lacking
86.72	2−	86.66	6	
3690.30	1	3690.24	6	
3702.67	2	3702.65	6	Separated. λ *Mo* > λ *Fe*. Rowland gives λ *Fe* 02.63
08.73	2−	08.74	6	Separated. λ *Mo* < λ *Fe*
18.66	1+	18.57	4	Separated. λ *Mo* > λ *Fe*
27.86	3	27.80	3	Separated. Δλ=0.09. Rowland has 27.78
3796.19	1.2	3796.14	6	*Fe* line lacking

Mo λ	*Mo* *i*	*Fo* λ	*Fo* *i*	Remarks
3801.14	1	3801.18	5	Trace of *Fe*, but not observable
02.00	2.3	01.95	6	Coinc.? λ *Mo* Δ λ *Fe*? Rowland gives λ *Fe* 01.95
06.15	2	06.15	6	*Fe* line lacking
10.99	1	10.92	4	Separated. λ *Mo* > λ *Fe*. Rowland has 10.90 *Fe*
14.64	1+	14.69	4	Widely separated
18.83	2−	18.80	6	*Fe* line lacking
21.82	1	21.75	6	*Fe* line lacking
24.34	1.2	24.27	6	Separated. λ *Mo* > λ *Fe*
25.63	1+	25.57	6	Coinc.? Perhaps λ *Mo* > λ *Fe*
29.04	3−	29.05	6	*Fe* line lacking
29.95	1.2	29.89	5	Not separable. Rowland gives ⊙ 29.91 but no
30.98	2−	30.98	5	*Mo* line between two *Fe* lines [*Fe* line
44.09	1.2	44.11	6	Separated. λ *Mo* < *Fe*
48.45	2+	48.45	6	Separated. λ *Mo* < *Fe*
64.25	10	64.19	6	Coinc.? *i Fe* > 6
3893.50	1+	3893.50	4	Separated. Rowland has 93.54
3903.07	10	3903.11	2	Coinc.? λ *Mo* < λ *Fe*?. Both lines reversed
71.54	1.2	71.46	3	Separated. λ *Mo* > λ *Fe*
73.10	1+	73.05	6	Separated. λ *Mo* > λ *Fe*
3974.09	2	74.15	6	Separated
4000.67	2	4000.65	5	Separated. Rowland gives ⊙ 00.61 *Fe*. This line lies between *Mo* 00.67 and 00.55
06.85	1	06.79	5	Separated
20.59	1.2	20.62	6	Separated. λ *Mo* < *Fe*
32.65	1.2	32.62	6	*Fe* line uncertain
56.18	2	56.12	6	Coinc.? Trace of *Fe* line. *Mo* line between 56.22 *Cr* and 56.13 *Fe*
76.35	2	76.40	6	Separated. λ *Mo* < λ *Fe*. R. has ⊙ 76.38 *Fe*
86.16	2	86.14	6	Coinc.?
4093.32	1+	4093.36	6	*Fe* line lacking
4105.27	2+	4105.35	6	*Fe* line lacking. Rowland has ⊙ 05.32. The lines are separated
07.63	3	07.65	2	Separated. λ *Mo* < λ *Fe*
08.30	1.2	08.30	6	Coinc.? *Fe* line faint and diffuse
10.46	1	10.48	6	*Fe* line lacking
15.08	2−	15.05	5	Coinc.? *Fe* line diffuse. Rowland has ⊙ 15.09
19.12	2	19.07	5	Separated. λ *Mo* > λ *Fe*
23.83	2	23.88	4	Widely separated. Rowland λ *Fe* = 23.91
29.02	2−	28.98	6	?
35.55	1+	35.50	6	*Fe* line lacking
37.10	1+	37.13	2	Widely separated. Rowland λ *Fe* = 37.16
52.07	2−	52.11	5	Separated. Δλ > 0.04
57.59	2.3	57.53	6	*Fe* line lacking
60.44	1+	60.38	6	*Fe* line lacking
70.01	2−	69.97	6	Coinc.? *Fe* line diffuse
70.55	1	70.49	6	*Fe* line lacking
78.72	1	78.71	6	Perhaps separated and λ *Mo* > λ *Fe*
80.69	1	80.67	6	*Fe* line lacking
81.24	2	81.23	6	*Mo* line dupl. — *Fe* line coincides with the violet component
84.33	1	84.38	6	*Fe* line lacking
4190.17	2−	4190.14	6	Trace of *Fe*. λ *Mo* > λ *Fe*? K. and R. assign the line to *Mn*
4200.02	1	4200.03	6	*Fe* line lacking

Mo λ	*i*	*Fe* λ	*i*	Remarks
4201.35	1+	4201.33	6	*Fe* line lacking
42.97	1.2	42.87	5	Separated
46.19	2.3	46.20	4	Separated
58.85	1+	58.77	5	*Fe* line concealed by a ghost
73.23	1.2	73.18	6	*Fe* line lacking
4277.38	3	4277.36	6	Trace of *Fe*. Coinc.?
4310.58	2−	4310.57	6	Separated. λ *Mo* > λ *Fe*
24.72	1	24.71	6	*Fe* line lacking
50.53	3−	50.48	6	*Fe* line lacking
69.23	2.3	69.23	6	*Fe* line lacking
75.07	1.2	75.11	6	*Fe* line lacking. K. and R. assign the line to *Mn*
75.21	1.2			
91.71	1.2	91.73	6	Separated.? The *Fe* line belongs to *Co*
4396.83	2	4396.83	6	Trace of *Fe* line. Coinc.?
4403.07	2+	4403.02	6	*Fe* line not visible, concealed by a ghost.
24.40	1	24.33	6	Separated
26.86	2.3	26.81	6	Separated. Δλ > 0.05
37.06	2−	37.11	5	Separated. λ *Mo* < λ *Fe*
44.21	1+	44.22	6	*Fe* line lacking
46.62	3−	46.54	6	*Fe* line concealed by a ghost
64.96	3−	64.95	4	*Mo* line on the red edge of the *Fe* line
68.46	3	68.51	6	*Fe* line lacking
4489.17	1.2	4489.15	6	Separated. λ *Mo* > λ *Fe*
4515.36	2−	4515.43	6	Separated
18.61	1+	18.69	6	Separated
28.77	2.3	28.85	1	Probably separated; lines broad and hard to separate
35.00	2+	35.01	6	*Fe* line lacking
60.32	2+	60.33	5	Coinc.? *Mo* line sharp, *Fe* line diffuse
79.92	1	80.00	6	Separated
4582.52	1+	4582.58	6	*Fe* line lacking
4611.36	2−	4611.45	2	Widely separated
26.67	3.4	26.72	6	Separated. λ *Mo* < λ *Fe*
27.70	2.3	27.72	6	Probably separated
30.20	2−	30.29	4	Widely separated
41.12	1−	41.19	6	Separated. λ *Mo* < λ *Fe*
51.25	2+	51.34	4	Widely separated
62.11	2.3	62.16	5	Separated. λ *Mo* < λ *Fe*
63.31	1+	63.32	6	Separated. λ *Mo* < λ *Fe*
4688.41	2.3	4688.46	6	Separated. λ *Mo* < λ *Fe*. *Fe* line broad, diffuse
4707.44	3.4	4707.52	2	Separated. λ *Mo* < λ *Fe*
31.64	3.4	31.67	6	Separated. λ *Mo* < λ *Fe*
34.34	1+	34.32	6	Separated. λ *Mo* > λ *Fe*
40.58	1	40.55	6	Separated. λ *Mo* > λ *Fe*
4749.61	1−	4749.56	6	*Fe* line lacking
4808.29	2−	4808.32	6	Separated
11.28	2.3	11.29	6	Coinc. Belongs to *Mo*
17.92	2−	17.97	6	Separated
4860.99	1	4860.99	6	*Fe* line lacking
4957.78	3	4957.87	2	Separated. Rowland gives for the *Fe* line 57.78, probably should be 57.88.
4964.63	2−	4964.72	6	Separated
5047.90	2	5047.92	6	*Fe* line lacking
5109.90	2+	5109.82	6	Separated

Mo λ	*i*	*Fe* λ	*i*	Remarks
5115.86	1—	5115.94	6	*Fe* line lacking
5232.58	1—	5232.55	6	*Fe* line lacking
5394.75	2	5394.81	6	Separated. λ *Mo* < λ *Fe*
5426.24	1	5426.21	6	Separated. λ *Mo* > λ *Fe*
5533.26	6	5533.17	5	Separated
34.85	1	34.94	6	*Fe* line lacking
5543.38	2+	5543.41	4	Coinc.? *Mo* line sharp. *Fe* line diffuse
5651.54	1+	5651.60	6	*Fe* line lacking
5672.35	1+	5672.39	6	*Fe* line lacking
5707.26	1.2	5707.22	5	Coinc.? Belongs to *V*
08.28	1+	08.32	5	Separated. λ *Mo* < λ *Fe*
12.05	2	12.09	5	Separated. λ *Mo* < λ *Fe*. K. and R. assign the line to *Ti*
27.30	2	27.27	6	Coinc.? Perhaps λ *Mo* > *Fe*
5771.33	1+	5771.35	6	*Fe* line lacking
5849.16	1.2	5849.14	6	*Fe* line lacking

In my former comparisons with the iron spectrum of Kayser and Runge I added the correction + 0.07 tenth-meters in order to render their wave-lengths, which are based on Bell's earlier determinations for D_1, comparable with my measures, which are referred to the later system of Rowland. Nevertheless, I had at that time repeated occasion to doubt the accuracy of this correction, and this suspicion has been fully confirmed by a comparison of Kayser's recently issued list of standards in the arc spectrum of iron with the former measurements. If we group the differences appearing in such a comparison in sections of 100 tenth-meters and take the means, we obtain the following values of the correction:

λ 3400 —	λ 3500	Δλ = + 0.07 tenth-meters
3500 —	3600	.04
3600 —	3700	.01
3700 —	3800	.02
3800 —	3900	.03
3900 —	4000	.05
4000 —	4100	.08
4100 —	4200	.07
4200 —	4300	.02
4300 —	4400	.05
4400 —	4500	.06

We see that the correction varies considerably in the different parts of the spectrum. On applying these new values to the

earlier measures of Kayser and Runge I was able to bring them into a far better agreement with my observations than formerly. Thus, for instance, cases occurred where at a very close coincidence of an iron line with a line of a different metal the use of the former correction led actually to a precisely opposite relative position of the two lines — the iron line being thus transposed, say, to the red edge of the other line, while on the double exposure of the two spectra just the reverse position could be distinctly observed. With the aid of the above corrections such contradictions have been disposed of almost without exception. On account of the small differences which still exist between the Kayser-Rowland iron standard lines, on the one hand, and the list of solar lines by Rowland, on the other, the comparison of my observations of the molybdenum lines with Kayser and Runge's iron lines cannot be carried out with the greatest rigor, but this is, however, sufficiently sharp for the portion of spectrum under consideration to render the results of the above table adequately reliable. For the remaining portion of the spectrum, between D and $\lambda 4500$, I have, of course, been obliged to employ the old correction.

We see from the table that in the great majority of approximate coincidences between the lines of iron and molybdenum the former are either lacking in my spectrum, or, if present, are clearly separated from the latter and consequently are independent. I would therefore express no definite opinion at present whether this absence of a number of the faintest lines of Kayser and Runge's iron spectrum is solely the consequence of a lower intensity of current employed by me, or indicates a greater purity of my sample of iron; but a certain probability is lent to the latter supposition by the fact that a quantity of impurities are actually present in the iron spectrum of Kayser and Runge. Moreover, since the cases are very rare in which a coincidence could be demonstrated without ambiguity between lines of molybdenum and the Sun, it was to be expected that the approximate coincidences with iron lines would in many cases prove only apparent, while, on the other hand, in the comparatively few doubtful cases such an explanation gains in proba-

bility. This is still more the case, since in such more or less certain coincidences the lines show a different character, those of molybdenum being always sharp, while those of iron are frequently diffuse.

MOLYBDENUM AND CHROMIUM.

The following table contains the approximate coincidences between lines of molybdenum and of chromium obtained by a comparison with my former measures of the latter spectrum; as will be seen, the investigation of the double exposures has shown these coincidences to be only apparent, almost without exception:

Mo λ	*i*	*Cr* λ	*i*	Remarks
3548.88	1	3548.95	1.2	Separated. *Cr* line very diffuse
64.45	1.2	64.44	1	Coinc. Belongs to *Mo*
66.20	2+	66.23	2.3	Coinc.? *Cr* line diffuse
3584.42	1.2	3584.45	2.3	Separated? λ *Mo* < λ *Cr*. *Cr* line diffuse
3603.86	1.2	3603.86	2	Coinc.? *Cr* line diffuse
08.52	2+	08.52	1.2	Coinc.
12.62	2—	12.70	1.2	Separated
13.80	1.2	13.78	1.2	λ *Mo* < λ *Co*. *Cr* line diffuse
81.88	2—	3681.81	1.2	λ *Mo* > λ *Cr*
43.98	1+	44.01	3	λ *Mo* < λ *Cr*
44.55	2—	44.63	2	Separated
47.37	2—	47.40	1.2	λ *Mo* < λ *Cr*
48.66	2+	48.73	2	
56.02	1.2	55.97	1	Coinc. Belongs to *Mo*?
68.78	1+	68.85	2	Separated. The *Cr* line coincides with *Mo* 68.92 ∴ λ *Cr* too small
3797.20	1.2	3797.28	2	
3808.04	1	3808.06	2	λ *Mo* < λ *Cr*
22.14	1	22.22	1	
25.50	1.2	25.54	2	The *Cr* line is between *Mo* 25.63 and 25.50
30.22	2—	30.17	3.4	Separated λ *Mo* > λ *Cr*
34.82	1.2	34.88	2.3	
3886.98	2.3	3886.94	3	λ *Mo* > λ *Cr*
3903.07	10	3903.02	3	λ *Mo* > λ *Cr*. Rowland gives 03.09 *Fe*, *Cr*
17.09	2—	17.15	1—	λ *Mo* < λ *Cr*
20.25	1—	20.25	1.2	Coinc.? Belongs to *Co*?
28.86	1.2	28.79	4	Separated. λ *Mo* > λ *Cr*
69.17	1	69.20	2	Separated. λ *Mo* < λ *Cr*
3994.06	2—	3994.10	1.2	Separated. λ *Mo* < λ *Cr*
4025.64	1.2	4025.60	1	Separated. λ *Mo* > λ *Cr*
43.91	1.2	43.85	1.2	Separated. λ *Mo* > λ *Cr*
56.18	2—	56.17	1.2	Separated. λ *Mo* < λ *Cr*. λ *Mo*=56.22 according to Rowland
67.88	1+	67.94	1	Widely separated

λ *Mo*	*i*	λ *Cr*	*i*	Remarks
4081.94	2—	4081.88	1	Coinc. λ *Cr* erroneous
4142.28	1+	4142.31	1.2	Separated. λ *Mo* < λ Cr
75.32	1	75.34	1.2	
4186.47	2—	4186.50	1.2	
4284.77	3	4284.84	1.2	Widely separated
4338.90	2—	4338.95	1.2	λ *Mo* < λ *Cr*
63.21	1—	63.25	2.3	λ *Mo* < λ *Cr*
76.87	1+	76.95	2	Separated. λ *Mo* < λ *Cr*
80.80	1.2	80.73	1	λ *Mo* > λ *Cr*
4397.48	2+	4397.40	2	λ *Mo* > λ *Cr*
4424.40	1	4424.40	2.3	Separated. λ *Mo* < λ *Cr*. Δλ=0.07. λ *Cr* erroneous; Rowland gives 24.46
42.37	2.3	42.43	1.2	Separated. λ *Mo* < λ *Cr*
4458.84	1.2	4458.75	2.3	Widely separated
4526.56	2+	4526.65	4	Widely separated
41.75	2—	41.70	2	Separated. λ *Mo* > λ *Cr*
86.25	1+	86.31	2	Separated. λ *Mo* < λ *Cr*
4588.33	1.2	4588.38	1.2	Separated. λ *Mo* < λ *Cr*
4610.07	3	4610.07	1.2	Coinc. Belongs to *Mo*
14.94	1	14.92	1.2	Separated? λ *Mo* > λ *Cr*
48.02	2	48.00	1.2	Separated. λ *Mo* < λ *Cr*
56.57	1	56.61	1.2	Separated. λ *Mo* < λ *Cr*
4673.24	1	4673.30	1.2	Widely separated
4700.71	2—	4700.77	2	?
06.25	2	06.25	1.2	Coinc. Belongs to *Mo*
23.27	1.2	23.28	2.3	Separated. λ *Mo* < λ *Cr*. Δλ exceedingly small
4783.16	2.3	4783.16	1.2	Separated. λ *Mo* < λ *Cr*. *Cr* line diffuse

MOLYBDENUM AND COBALT.

As in the previous cases, the lines of approximately equal wave-lengths are here again practically wholly independent of each other, as is seen from the following table. In a few cases where a separation could not be effected, the molybdenum line is commonly too strong to be considered as impurity. In the region from D to F there occurs only a single pair of lines whose components differ from each other by less than 0.05 tenth-meters, namely, *Mo* 5431.27 and *Co* 5431.30, and this pair was not specially investigated.

Mo λ	*i*	*Co* λ	*i*	Remarks
3562.26	2−	3562.22	2.3	Somewhat separated. λ *Mo* > λ *Co*
3582.03	3.4	3582.00	2	Coinc.
3605.19	1+	3605.19	2	Coinc. Foreign Line?
08.52	2+	08.50	1.2	Separated. λ *Mo* > λ *Co*
39.71	1+	39.63	2.3	Widely separated
51.48	2	51.42	2	Widely separated
86.72	2−	86.63	1.2	Widely separated
3693.52	2−	3693.53	1.2	Coinc. Foreign Line?
3702.33	1+	3702.40	2.3	Separated λ *Mo* < λ *Co*
33.59	1+	33.62	2.3	Separated λ *Mo* < λ *Co*
3759.80	1.2	3759.83	1.2	Separated λ *Mo* < λ *Co*
3812.63	2	3812.57	1.2	Separated λ *Mo* > λ *Co*
14.64	1+	14.58	2	Coinc.?
19.98	2.3	20.02	2	λ *Mo* < λ *Co*
66.87	1+	66.92	1	λ *Mo* < λ *Co*
73.30	1.2	73.25	4.5	λ *Mo* > λ *Co*
3893.50	1+	3893.44	1.2	Separated. λ *Mo* > λ *Ci*
3945.41	2−	3945.47	3	
47.33	2−	47.28	1.2	λ *Mo* > λ *Co*
66.52	2−	66.52	3	Coinc. *Mo* line too strong to come from *Co*
69.17	1	69.25	2.3	
3976.35	2	3976.28	2.3	?
4076.69	1.2	4076.74	2	Separated. λ *Mo* < λ *Co*
4207.75	1+	4207.77	1.2	Separated. λ *Mo* < λ *Co*
68.25	2−	68.18	1.2	λ *Mo* > λ *Co*
4292.34	3	4292.41	2	
4375.07	1+	4375.09	2.3	Separated λ *Mo* < λ *Co*
4391.71	1.2	4391.70	3	λ *Mo* < λ *Co*=91.74?
4490.37	2	4490.46	1.2	
4492.24	1	4492.23	1.2	λ *Mo* < λ *Co*
4517.30	3+	4517.28	3.4	Coinc. Impossible to separate. Rowland gives 17.32
4553.52	1.2	4553.51	1.2	λ *Mo* < λ *Co*
4651.25	2−	4651.28	1.2	λ *Mo* < λ *Co*
82.44	1−	82.53	4	
4686.01	2−	4686.05	1.2	Separated. λ *Mo* < λ *Co*
4776.54	3	4776.49	3.4	λ *Mo* > λ *Co*
85.34	2.3	85.26	2.3	*i Co* < 2.3
4792.96	2	4793.03	4	

MOLYBDENUM AND NICKEL.

The comparison of the spectra of these two elements has yielded only a small number of approximate coincidences, as appears from the following table. Since most of these are not real we may consider the molybdenum to be practically free from nickel.

Mo λ	*i*	*Ni* λ	*i*	Remarks
3533.87	2	33.89	1	*Cu* 33.90
3566.57	1	66.50	4.5	
3793.75	2−	93.75	3	
3994.06	2−	94.13	2	These lines have not been specially investigated
4006.23	2	06.30	2	
4075.72	2−	75.75	1.2	
4138.72	1.2	38.67	1	
4184.59	1.2	84.65	1.2	
4284.77	3	84.83	2.3	Widely separated
4325.44	1	25.49	1.2	Separated? λ *Mo* > λ *Ni*?
4423.24	1+	23.24	1.2	Coinc.
4553.40	1.2	53.37	1.2	Separated. λ *Mo* > λ *Ni*
4567.57	1−	67.59	1	Separated?
4618.15	1+	18.22	1.2	Coinc. Impurity?
4786.67	2	86.66	3	Rowland gives ⊙ 86.73 *Ni*. Separated
4792.96	2	92.98	1	Separated. λ *Mo* < λ *Ni*
4817.92	2−	17.97	1	Widely separated
5058.30	1	58.22	1	Separated
5080.23	2.3	80.16	1.2	Separated
5642.05	1−	42.08	1.2	Separated. λ *Mo* < λ *Ni*

MOLYBDENUM AND MANGANESE.

The following table shows that here also there were only a few isolated cases in which a separation of close pairs of lines in

Mo λ	*i*	*Mn* λ	*i*	Remarks
3669.50	2.3	3669.54	1	Separated. λ *Mo* < λ *Mn*
3680.36	1+	3680.32	1	Separated. λ *Mo* > λ *Mn*
3763.52	2	3763.51	2−	Coinc.?
67.90	1+	67.84	2−	Separated. λ *Mo* > λ *Mn*
3776.73	1+	3776.70	1.2	Probably separated and λ *Mo* > λ *Mn*
3830.08	1+	3830.12	1+	Separated. λ *Mo* < λ *Mn*.
33.92	3	33.96	3.4	Separated. λ *Mo* < λ *Mn*.
44.09	1.2	44.10	3.4	Separated. λ *Mo* < λ *Mn*. Δλ > 0.01
3896.55	1.2	3896.48	1.2	Separated
3908.42	1.2	3908.34	1	Widely separated
11.24	1.2	11.27	1.2	Separated. λ *Mo* < λ *Mn*
3936.89	1+	3936.91	1+	Coinc. Rowland gives *Mn*
4008.21	1+	4008.19	1.2	Perhaps λ *Mo* > λ *Mn*
4075.43	2	4075.39	1.2	Coinc.
4132.41	2−	4132.45	1	Separated? *Mo* line faint. ∴ *i Mo* var. Really *Mo*?
4148.88	1+	4148.94	2.3	Widely separated
4235.23	1.2	4235.28	3	λ *Mo* < λ *Mn*
4375.07	1+	4375.10	2	λ *Mo* < λ *Mn*
4381.82	4	4381.87	2−	Separated. λ *Mo* < λ *Mn* *Mn* line lies between *Mo*: 75.21 and .07
4452.77	1.2	4452.73	1.2	λ *Mo* < λ *Mn*
4586.25	1+	4586.30	1+	λ *Mo* > λ *Mn*
4626.67	3.4	4626.74	2	Separated
4838.35	1	4838.40	1	

the two spectra could not be effected. In view of the not inconsiderable intensity of these lines the assumption of an impurity by a third metal seems hardly probable.

MOLYBDENUM AND TITANIUM.

In the region of spectrum from D to F there is but a single pair whose components have very nearly the same wave-length, namely, *Mo* 5712.05 and *Ti* 5712.07. These are, however, independent of each other, since the *Mo* line lies at the edge of the solar line at 5712.09, which is certainly identical with the *Ti* line. The line is ascribed to iron by Rowland as well as by Kayser and Runge. The remaining pairs of lines in approximate agreement are given in the following table, from which it again appears that in almost every case the lines are independent of each other.

Mo λ	*Mo* *i*	*Ti* λ	*Ti* *i*	Remarks
3493.49	2—	3493.44	1	Separated. λ *Mo* > λ *Ti*
3510.93	1.2	3510.98	3	Widely separated
3566.20	2+	3566.16	1.2	Separated. λ *Mo* > λ *Ti*
3613.94	1.2	3613.89	2	
14.42	3	14.35	2	Separated
35.30	2	35.33	2	Separated. λ *Mo* < λ *Ti*
35.57	2.3	35.61	4.5	Separated. λ *Mo* < λ *Ti*
36.07	1	36.05	1	Coinc. Impurity?
54.73	2+	54.72	3	Nearly coinc., but λ *Mo* < *Ti*
62.30	1.2	62.37	2.3	Widely separated
63.83	1+	63.82	1	Coinc. Common impurity?
77.83	2—	77.90	1	Separated, but λ *Mo* > λ *Ti* = 77.80?
3688.12	1	3688.19	1	*Ti* line lacking
3702.33	1+	3702.42	2	Widely separated
35.80	1.2	35.84	1.2	Separated. λ *Mo* < λ *Ti*
61.93	2—	62.01	1	Widely separated
66.58	1	66.60	1	Separated. λ *Mo* < λ *Ti*
76.27	1+	76.20	1	Separated. λ *Mo* > λ *Ti*
88.42	2	88.44	1.2	*Ti* line lacking
3798.39	10	3798.47	1.2	Separated
3806.15	2	3806.19	1	*Ti* line lacking
14.56	1.2	11.56	1	Coinc. *Mo?*
14.64	1+	14.72	2	Widely separated
21.82	1	21.86	1	*Ti* line lacking
22.14	1	22.16	2.3	Separated. λ *Mo* < λ *Ti*
29.95	1.2	29.87	1.2	Separated
48.45	2+	48.48	1.2	Probably separated. λ *Mo* < *Ti*
74.34	1.2	74.32	2	Coinc. *Ti* line faint
3888.15	1—	3888.20	2	Separated. λ *Mo* < λ *Ti*. *Ti* line diffuse
3913.52	1.2	3913.58	2.3	Separated. λ *Mo* < λ *Ti*

Mo λ	*i*	*Ti* λ	*i*	Remarks
3934.41	1.2	34.37	1.2	Separated. λ *Mo* > λ *Ti*
3994.79	1	3994.84	1.2	Separated. λ *Mo* < λ *Ti*
4008.21	1+	4008.20	2	Coinc. *Mo* line sharp. *Ti* line diffuse
34.11	1	34.05	1.2	Separated. λ *Mo* > λ *Ti*
4057.77	1.2	4057.76	1.2	Coinc. Impurity?
4105.27	2+	4105.31	1.2	Separated. λ *Mo* < λ *Ti*. The line 05.31 belongs to *V*
64.26	1.2	64.27	1.2	Separated. λ *Mo* < λ *Ti*
4166.47	1.2	4166.45	2	Separated, but λ *Mo* < λ *Ti*
4224.93	1	4224.96	2	Separated. λ *Mo* < λ *Ti*
60.85	1.2	60.91	1	Separated
66.37	2—	66.37	1.2	Separated. λ *Mo* < λ *Ti*
80.17	1+	80.17	1.2	Separated. λ *Mo* < λ *Ti*. Δλ = 0.04
4291.39	2—	4291.32	2	Separated. λ *Mo* > λ *Ti*
4335.00	2	4334.98	1.2	Separated, but λ *Mo* < *Ti*
4417.40	1	4417.46	3	Separated. λ *Mo* < λ *Ti*
33.68	1.2	33.75	1.2	Widely separated
57.55	3.4	57.59	3.4	Separated. λ *Mo* < λ *Ti*
4489.17	1.2	4489.24	2.3	Separated. λ *Mo* < λ *Ti*
4501.44	2	4501.43	3	Separated? λ *Mo* > λ *Ti*?
35.00	2+	34.97	3.4	Separated. λ *Mo* > λ *Ti*
58.30	2.3	58.28	1.2	Coinc. *Mo*?
4599.35	2—	4599.40	2	Separated
4656.57	1	4656.60	3.4	Separated. λ *Mo* < λ *Ti*
4723.27	1.2	4723.32	2.3	
4808.68	1+	4808.70	2	
4811.28	2.3	4811.24	2	

MOLYBDENUM AND VANADIUM.

In the spectra of these two metals I have met with lines of the same, or nearly the same wave-lengths, in only a few isolated cases, which might be ascribed to the two metals on the basis of their ratios of intensity. The pairs are these:

Mo λ	*i*	*V* λ	*i*
3524.35	2+	3524.38	2+
4240.26	2	4240.25	2
4776.54	3	4776.54	3
5241.09	3	5241.06	2

As for the rest, the following table shows that the approximate coincidences found are not real.

After these comparisons with the arc spectra of the metals investigated by myself, I now proceed to the comparison of my

Mo λ	*i*	*V* λ	*i*	Remarks
3498.21	1+	3498.23	1	*V* line lacking
3504.55	2.3	3504.57	1.2	Separated ?
24.35	2+	24.38	2+	Coinc. ?
3562.26	2−	3562.32	1	Separated
3638.57	1.2	3638.57	1	Coinc. Foreign line ?
3669.50	2.3	3669.57	1.2	Widely separated
3732.91	3	3732.88	2	*Mo* line dupl.
34.56	1.2	34.59	1.2	Separated. $\lambda Mo < \lambda V$
70.66	2.3	70.68	1	*V* line lacking
75.82	1+	75.85	1.2	Separated. $\lambda Mo < \lambda V$
3776.27	1+	3776.31	1.2	Separated. $\lambda Mo < \lambda V$
3822.14	1	3822.14	2.3	Slightly separated. $\lambda Mo < \lambda V$
47.41	2+	47.46	2.3	Separated. $\lambda Mo < \lambda V$
70.77	1.2	70.72	1+	*V* line faint. λMo seems $> \lambda V$
3888.15	1−	3888.23	1+	Separated
3916.62	1+	3916.55	1.2	Separated
24.78	1	24.84	2.3	Coinc. close
31.57	1.2	31.50	2	Separated. $\lambda Mo > \lambda V$
35.33	1.2	35.28	2.3	
3950.40	1	3950.37	2−	
4003.62	1+	4003.70	1.2	
05.86	1	05.86	2+	Coinc. *V*? Other strong *V* lines are lacking in *Mo*
32.65	1.2	32.62	1.2	Separated. $\lambda Mo < \lambda V$
4067.88	1+	4067.90	1.2	Separated. $\lambda Mo < \lambda V$
4102.33	2.3	4102.32	3	Slightly separated. $\lambda Mo < \lambda V$
05.27	2+	05.32	3	Separated
07.63	3	07.64	1.2	Slightly separated. $\lambda Mo < \lambda V$
08.30	1.2	08.36	2	Widely separated
32.07	2−	32.13	3.4	Separated. *V* line reversed
4175.32	1	4175.30	1	Coinc. Foreign line ?
4240.26	2	4240.25	2	Coinc.
40.48	2	40.53	2+	Separated. $\lambda Mo < \lambda V$
77.08	3	77.12	3	Separated
79.19	1+	79.12	1.2	
91.39	2	91.46	2−	
4296.35	1.2	4296.28	2.3	
4369.23	2.3	4369.25	1+	Hard to separate. $\lambda Mo < \lambda V$?
4392.32	1.2	4392.24	2	Separated. $\lambda Mo < \lambda V$
4506.22	3	4506.30	2	Separated very slightly $\lambda Mo < \lambda V$ $\lambda Mo < \lambda V$
4611.03	1+	4611.10	1.2	
26.67	3.4	26.67	2+	
30.20	2−	30.24	1	
4648.02	2	4648.08	1	
4706.25	2	4706.34	2.3	Widely separated
76.54	3	76.54	3	Coinc.
84.64	1	84.65	2	$\lambda Mo < \lambda V$
4786.68	2	4786.70	3	Separated. $\lambda Mo < \lambda V$
4833.13	1.2	4833.17	2	$\Delta\lambda > 0.04$
5014.80	1+	5014.83	3	Separated. $\lambda Mo < \lambda V$
5241.09	3	5241.06	2	Coinc. $i\,V < 2$
5437.97	2.3	5437.93	1.2	Separated. $\lambda Mo > \lambda V$
5632.74	4	5632.73	1+	Coinc ? Perhaps $\lambda Mo < \lambda V$
5734.32	2−	5734.26	2	Coinc ? *V*?
5747.93	1+	5747.98	1	Separated. $\lambda Mo < \lambda V$

observations with the arc spectra of other metals investigated by Kayser and Runge. For this purpose I first made out accurate lists of all those lines of molybdenum which agreed within the limits of error given by Kayser and Runge with lines measured by them. But nothing could be concluded from the approximate agreement of the positions, since these limits of error exceed from ten to a hundred times the uncertainty attaching to my determination of the molybdenum lines, on account of the diffuseness of the lines of lithium, sodium, potassium, caesium, antimony, mercury, copper, silver, and gold. For the present I therefore regard my spectrum of molybdenum as free from impurities due to these metals. This also holds good for the metals rubidium, magnesium, zinc, aluminium, indium, arsenic, thallium, and tin, for which no lines agreeing with molybdenum occur in the tables of Kayser and Runge. The remaining metals investigated by Kayser and Runge—calcium, strontium, cadmium, barium, lead, and bismuth—present in their arc spectra only isolated cases of approximate coincidence with the lines of molybedenum, as to the actual nature of which I can at present give no definite decision, In the summary of these lines given in the following table I have only taken into account those in which the limit of error assigned as possible by Kayser and Runge does not exceed the amount of 0.05 tenth-meters, since for greater values of this limit the close agreement of the wave-lengths is in my opinion too likely to be accidental.

	Kayser and Runge				Hasselberg	
	λ	Limits of Errors	*i*	Remarks	λ *Mo*	*i*
Bi	3888.34	0.03	6		88.36	2—
Pb	4062.30	0.03	4	reversed	62.24	2.3
Ba	3566.90	0.05	5	diffuse	66.91	1
	4224.11	0.05	4		24.10	1
	4291.32	0.05	4		91.39	2—
	4325.38	0.05	5		25.44	1
	4402.75	0.05	2	reversed	02.67	2—
	4407.10	0.05	4	diffuse toward red	07.04	1+
	4700.64	0.05	3	diffuse toward red	00.71	2—
Ca	3973.89	0.05	3	diffuse toward red	73.92	2+

The first of these lines may well belong to calcium, although it seems less probable on account of the diffuseness on one side of the lines in the spectrum of this metal. The first five of the barium lines should be separated from the corresponding molybdenum lines on account of the very considerable difference of the wave-lengths, while the last two should perhaps be removed from the molybdenum spectrum. I have, however, not removed the first or the last two lines of this table from the catalogue of molybdenum lines.

It now finally remains to compare the spectrum of molybdenum with the arc spectra of the metals of the platinum group recently investigated by Kayser. A considerable number of approximate coincidences was to be expected in advance on account of the extraordinary abundance of lines in these spectra, even if on account of the high accuracy of Kayser's measures the comparisons are limited to those cases in which the difference of wave-lengths does not exceed 0.05. This limitation appears fully justifiable, since in general Kayser's wave-lengths should be accurate to 0.01 and mine to 0.02 tenth-meters. I have collected the lines found to be of nearly the same wave-lengths in this comparison with the observations of Kayser, in the following table, in which I have reduced to my scale as well as I could his estimates of intensity, which range from 0 to 10:

	λ	*i*	*Mo* λ	*i*	Probably belongs to
Pt	3818.83	3	18.83	2—	*Pt*
	3900.87	2.3	00.87	1	*Pt*
	4081.63	1—	81.62	3	*Mo*
	4201.37	1+	01.35	1+	?
	4269.41	1+	69.44	2.3	*Mo*
	5306.49	1—	06.49	1—	?
Pd	4170.00	3	70.01	2—	*Pd*
	4344.8	2.3	44.88	1.2	?
	4443.19	2	43.23	2—	?
	4458.79	1.2	58.84	1+	?
	4632.77	1	32.75	1+	?
	5117.16	1.2	17.18	4+	*Pd*
	5619.67	2—	19.63	5.6	*Pd*
	5739.88	1	39.92	2.3	*Pd*

	λ	*i*	*Mo* λ	*i*	Probably belongs to
	3463.75	1− −	63.78		?
	3480.29	1+	80.26	1.2	?
	3482.50	1+	82.55	2	?
	3514.91	1−	14.93	1+	?
	3640.79	2.3	40.76	2+	*Ru, Mo*
	3686.74	1−	86.72	2−	*Mo*
	3702.37	1+	02.33	1+	?
	3716.32	2−	16.27	2	?
	3722.46	1−	22.50	1−	?
	3730.75	2−	30.75	1	?
	3733.19	1+	33.22	2−	?
	3742.44	3	42.48	2.3	*Ru, Mo*
	3744.55	1+	44.55	2−	?
	3764.18	1−	64.20	1	?
	3765.94	1− −	65.92	1.2	*Mo*
	3782.89	1− −	82.86	1	?
	3831.95	2.3	31.95	1+	*Ru*
	3911.28	2−	11.24	1.2	?
	3921.06	2.3	21.09	1+	*Ru*
Ru	3924.78	1+	24.78	1	?
	3939.27	1− −	39.30	1+	*Mo*
	4021.15	2−	21.19	2	?
	4032.65	1−	32.65	1.2	*Mo*
	4200.07	4+	00.02	1	*Ru*
	4207.80	1+	07.75	1+	?
	4245.00	2.3	44.96	1.2	*Ru*
	4287.21	2.3	87.26	2−	?
	4293.44	2.3	93.42	3	*Mo, Ru*
	4332.65	1+	32.68	1+	?
	4362.87	1−	62.87	1+	?
	4396.87	1− −	96.83	2	*Mo*
	4490.40	1+	90.37	2	?
	4617.83	1− −	17.82	1	?
	4833.16	1+	33.13	1.2	?
	5062.81	1−	62.76	1+	?
	5427.81	2.3	27.80	1	*Ru*
	5694.63	1+	94.64	1+	?
	5730.12	1+	30.17	2−	?
	5771.35	1+	71.33	1+	?
	3468.75	1+	68.70	1	?
	3476.18	1− −	76.15	2	*Mo*
	3628.84	3	28.80	1.2	*Ir*
	3641.04	1−	41.08	1	?
	3661.87	3	61.91	2+	*Ir, Mo*
	3747.35	2.3	47.37	2−	*Ir, Mo*
Ir	3768.82	1+	68.78	1+	?
	3800.24	1+	00.28	1+	?
	3817.38	1− −	17.37	1−	?
	4070.07	2.3	70.05	2+	*Ir, Mo*
	4200.03	1+	00.02	1	?
	4268.25	2.3	68.25	2−	*Ir, Mo*
	5364.51	1+	64.50	3.4	*Mo*

	λ	i	*Mo* λ	*Mo* i	Probably belongs to
	3490.46	1+	90.42	1	?
	3681.70	1+	81.69	1.2	?
	4003.65	1+	03.62	1+	?
	4124.76	1− −	24.72	2	*Mo*
	4201.54	1− −	01.50	1.2	*Mo*
Os	4233.63	1−	33.68	1.2	*Mo*
	4252.72	1− −	52.69	1+	*Mo*
	4294.10	1+	94.07	3	*Mo*
	4296.38	1− −	96.35	1.2	*Mo*
	4338.91	2	38.90	2−	?
	4692.22	1+	92.19	1+	?
	3469.35	1− −	69.39	2	*Mo*
	3469.77	3	69.80	1	*Rh*
	3612.62	3	12.62	2−	*Rh*
	3639.68	3.4	39.71	1+	*Rh*
	3651.52	1+	51.48	2	?
	3713.59	2−	13.64	2−	?
	3755.21	1−	55.31	2	*Mo*
	3765.23	3	65.21	1+	*Rh*
	3775.86	1+	75.82	1+	?
Rh	3812.60	1+	12.63	2	?
	3934.38	2.3	34.41	1.2	*Rh*
	3959.01	3	59.03	1	*Rh*
	4081.96	1+	81.94	2−	?
	4423.83	1−	23.79	2.3	*Mo*
	4506.82	1−	06.86	2	*Mo*
	4558.90	2.3	58.92	1.2	*Rh*
	4569.18	3.4	69.21	1+	*Rh*
	4608.29	1+	08.32	1	?
	5090.80	3	90.80	1	*Rh*
	5292.28	2.3	92.30	1.2	*Rh*
	5497.20	1− −	97.18	1.2	*Mo*
	5544.80	3.4	44.78	2.3	*Rh*, *Mo*

It appears hardly to be doubted from the large number of nearly coincident lines that on the one hand several of the faintest molybdenum lines of my catalogue are to be attributed to impurities of one or the other of the plantinum metals, while on the other hand several faint molybdenum lines occur in the spectra of the latter. On the assumption that the wave-lengths are actually identical, the origin of the lines can be given on the basis of the intensities with a pretty large degree of probability in about half the cases; but in the remaining cases where this basis of discrimination fails on account of nearly equal intensity there arises the possibility or indeed probability of contamination by a third metal. We see from the last column which

gives the probable origin of the lines according to this principle that of the molybdenum lines there are probably two due to platinum, four to palladium, five to ruthenium and ten to rhodium ; while there should be ascribed to molybdenum from platinum two, from ruthenium five, from rhodium five, from osmium five, and from indium two lines; and that finally in eight cases the lines may be simultaneously ascribed to two metals. All this, however, assumes an absolute coincidence. Since the decision of this question cannot be reached with entire rigor with the means at present at my disposal, in view of the exceedingly small quantities involved, I have preferred to provisionally retain the lines in my catalogue, in the possibility of investigating these details more accurately at some future time.

[*To be concluded.*]

THE SOLAR ATMOSPHERE

By ARTHUR SCHUSTER.

THE radiation received from different portions of the solar disk is known to diminish from the center toward the limb in a manner which is generally considered not to be consistent with the assumption of a uniformly absorbing solar atmosphere. The measurements recently published by Mr. Frank W. Very[1] allow us to examine this question in a more satisfactory manner than has hitherto been possible, and such examination leads to the conclusion that the difficulty which has been felt in explaining the law of variation of intensity across the Sun's disk, is easily removed. It is only necessary to place the absorbing layer sufficiently near the photosphere and to take account of the radiation which this layer, owing to its high temperature, must itself emit.

I introduce the following notation:

I = Intensity of the radiation, which is incident on the absorbing shell.

F = The radiation of a perfectly black body which is at the temperature of the shell.

A = the radiation leaving the absorbing layer in the direction of the Earth.

t = the length of path within the absorbing layer of rays which have traversed this layer.

t_0 = the thickness of the absorbing layer, *i. e.* the minimum value of t.

k = a constant on which the absorption depends, which may be different for different wave-lengths.

r = the perpendicular distance between any point on the Sun and the line drawn from the Sun's center towards the observer stationed on the Earth.

a = the Sun's radius.

Also for convenience of printing, write:

$$z = e^{-kt_0};\ \gamma = \frac{r}{a};\ \sigma = \frac{1}{\sqrt{1-\gamma^2}}\ .$$

The differential equation giving the law of variation of A with the distance t is:

$$\frac{dA}{dt} = -k(A - F)\ ,$$

[1] ASTROPHYSICAL JOURNAL, 16, 73, 1902.

and its appropriate solution is:

$$A = Ie^{-kt} + F(1 - e^{-kt}) ,$$

where

$$t = [(a + t_0)^2 - r^2]^{\frac{1}{2}} - [a^2 - r^2]^{\frac{1}{2}} .$$

If t_0 is small compared to a,

$$t = \frac{t_0}{\sqrt{1 - \gamma^2}} = \sigma t_0 ,$$

and the equation for the radiation may now be written:

$$A + (I - F) z^{\sigma} + F .$$

The three unknown quantities I, F, and z may be determined if the radiation is measured at three points of the solar disk. Let A_0, A_1, A_2 be these measured radiations and σ_0, σ_1, σ_2 the corresponding values of σ. By elimination of I and F we obtain:

$$(A_0 - A_1)(z^{\sigma_0 - 1} - z^{\sigma_2 - 1}) = (A_0 - A_2)(z^{\sigma_0 - 1} - z^{\sigma_1 - 1}) .$$

This determines z, which must be formed numerically by means of a series of systematic trials and interpolations. When z is determined I and F are calculated by the equations

$$I = \frac{A_0(1 - z^{\sigma_2}) - A_2(1 - z^{\sigma_0})}{z^{\sigma_0} - z^{\sigma_2}} ,$$

$$F = \frac{A_2 z^{\sigma_0} - A_0 z^{\sigma_2}}{z^{\sigma_0} - z^{\sigma_2}} .$$

If A_0 refers to the observation at the Sun's disk, it may be put equal to one, as in Mr. Very's observations, and I and F will then be referred to the same unit, which, however, it must be remembered will be different for different wave-lengths.

The solution of the equations may also be graphically carried out as follows:

For a particular value of z draw a family of curves satisfying the equation:

$$y = cz^{\sigma} ,$$

y being the ordinate, $\sigma = \sqrt{\frac{1}{1 - \gamma^2}}$ the abscissa and c a parameter which is varied step by step. A number of such families are drawn for different values of z. That curve is then chosen

which most nearly coincides with the curve of observed radiation, a displacement of the curve up and down being allowable. The amount of displacement necessary to bring the curves into coincidence will measure F, because y will for coincidence be equal to $A - F$. F being known, I will be determined from the corresponding relation $c = I - F$.

For convenience of reference I give in Table I Mr. Very's observations, on which the subsequent calculations are based. The wave-length for which the radiation is measured is denoted by λ and given in centimeters.

TABLE I.

λ	$\gamma = 0$	$\gamma = 0.5$	$\gamma = 0.75$	$\gamma = 0.95$
15.0×10^{-6}	Rad. taken as one	0.959	0.950	0.856
10.1	"	0.943	0.894	0.765
7.81	"	0.941	0.885	0.749
6.15	"	0.948	0.845	0.681
5.50	"	0.933	0.831	0.587
4.68	"	0.902	0.764	0.462
4.16	"	0.858	0.744	0.471
$\sigma =$	1	1.155	1.512	3.202

By combining the observed values for $\gamma = 0$; $\gamma = 0.75$ and $\gamma = 0.95$ I have calculated z, I, and F for different wave-lengths; the results are given in Table II.

TABLE II.

λ	z	I	F	$I - F$
15.0×10^{-5}	0.581	1.15	0.793	0.36
10.1	0.366	1.46	0.736	0.72
7.81	0.355	1.51	0.720	0.79
6.15	0.312	1.76	0.654	1.11
5.50	0.442	1.62	0.505	1.12
4.68	0.360	2.07	0.399	1.67
4.16	0.315	2.25	0.426	1.82

To obtain an idea of the accuracy with which the formula satisfies the observations, I have calculated with the values of Table II, the radiation for the position defined by $\gamma = 0.5$; the

observations at these points not having been made use of in the computation of the constants. The comparison is given in Table III.

TABLE III.

$10^6 \lambda =$	15.0	10.1	7.81	6.15	5.50	4.68	4.16
Calculated radiation	0.983	0.962	0.958	0.943	0.941	0.912	0.906
Observed radiation	0.959	0.943	0.941	0.948	0.933	0.902	0.858

The observed radiations seem to be generally smaller than the calculated ones, but the difference probably falls within the errors of observations. The five individual measurements which Mr. Very in his paper gives for one set of observations range from 0.908 to 0.959 (a difference of 5 per cent.) while, with the exception of the number for the shortest wave-length, when the measurements are admittedly doubtful, the difference between the calculated and observed radiations exceeds 2 per cent. in only one case.

The coefficient of absorption depends on the quantity of z, which is the fraction of the incident light passing normally through the absorbing layer. Table II shows no decisive indication that this quantity varies in a regular manner with the wave-length. No doubt the coefficient of absorption seems decidedly less for the longest wave-length, but a reference to Table I will show that the whole variation of intensity for this wave-length is so small that the coefficient of absorption calculated from it is very uncertain. Our equations show that constancy of radiation for different distances from the Sun's limb may be due either to smallness of the coefficient of absorption, in which case the radiation is everywhere equal to I, or to its greatness in which case the radiation is everywhere equal to F, or finally to the near equality of I and F. Table II shows that the difference between I and F diminishes considerably with increasing wave-lengths, and hence the coefficient of absorption has comparatively little influence on the results for the longest waves.

To confirm this, I have calculated the radiations transmitted for $\lambda = 15.0 \times 10^{-6}$, assuming a coefficient of absorption equal to its average value for the other wave-lengths, *i. e.*, $z = 0.358$.

The observations in that case may best be satisfied by taking $I = 1.249$, $F = 0.847$. The observed and calculated values now compare as follows :

	$\gamma=0.5$	$\gamma=0.75$	$\gamma=0.95$
Calculated radiation	0.970	0.932	0.862
Observed radiation	0.959	0.950	0.856

The disagreement of the numbers is not sufficient to allow us to state definitely that the coefficient of absorption has not the assumed value. That the absorption should be exactly the same for all wave-lengths is very unlikely, but the data we possess at present do not allow us to draw any conclusion as to the manner in which it varies.

I have confirmed in another manner that the available observations are not inconsistent with a constant coefficient of absorption. Assuming the constancy, the variation of the total radiation at different points of the Sun's disk should follow the same law as that of the separate portions of the spectrum. But the total radiation has been measured by Langley, and by Messrs. W. E. Wilson and Rambaut,[1] and the measurements being more easily made are probably more accurate than those of Mr. Very. Taking Mr. Wilson's observations and selecting again the same parts of the Sun's disk to determine the constants, *i. e.*, $\gamma = 1$, $\gamma = 0.75$, and $\gamma = 0.95$, I find that with $I = 1.486$, $F = 0.579$, $\varepsilon = 0.465$, the numbers given in Table IV are obtained.

[1] *Proceedings Royal Irish Academy*, **2**, No. 2, 1892.

TABLE IV.

$\gamma=$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Obser'd. rad.	0.999	0.996	9.988	0.973	0.953	0.925	0.887	0.819	0.749	0.451
Calcul'd rad.	0.999	0.994	0.985	0.973	0.954	0.927	0.890	0.832	0.736	0.579

With the exception of the radiation at the edge of the Sun which must be doubtful, the agreement is very close.

I return to the discussion of Table II. The justification of the view, that the radiation of the absorbing layer accounts for the discrepancy which has hitherto been found to exist between theory and observation, must be sought not so much in the

general agreement of the facts which is now arrived at (because our formula, having an additional constant to dispose of, can naturally be made to fit the facts better) as in the consistent fashion in which the calculated values of F give us numbers, such as we have *a priori* reason to expect. The quantity F is the radiation of a black body at the temperature of the absorbing layer. The curve of F drawn with the wave-length as abscissa should, if our supposition is correct, show a maximum, which, when compared to the radiation I of the photosphere, is displaced toward the red, and the highest ordinate should be less than the highest ordinate of I. To show that this is exactly what calculation gives us, I have made use of Professor Langley's observations, which give the observed intensity A_0 at the center of the disk for different wave-lengths. In the previous tables the value of A_0 was taken to be equal to 1 for each wave-length, and the values of I and F were therefore referred to a unit which changed with the wave-length. The second column in Table V gives the intensities of radiation for the wave-lengths given in the first column. The highest ordinate of the intensity curve is now taken to be the unit.

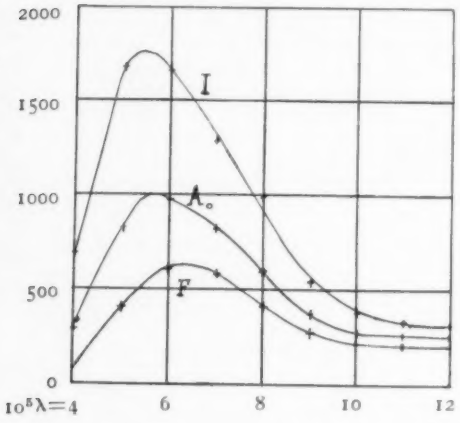
TABLE V.

$10^5 \times \lambda$	A^0	I	F
12	0.260	0.325	0.208
11	0.274	0.356	0.214
10	0.293	0.396	0.226
9	0.390	0.546	0.292
8	0.581	0.994	0.421
7	0.846	1.286	0.584
6	0.975	1.658	0.604
5	0.837	1.660	0.418
4	0.302	0.695	0.091

The third and fourth columns give the radiation (I) of photosphere before it traverses the absorbing layer and the radiation (F) of a black body at the temperature of the absorbing layer.

The numbers given in Table V are plotted in the figure, and show clearly that the radiation of the absorbing layer belongs to a lower temperature than that of the rays traversing it. The

maximum of the curve of I lies at about $\lambda = 5.4 \times 10^{-5}$ that of the curve of F at 6.4×10^{-5}. The ratio of these two numbers, or 0.84, should give the ratio of the absolute temperature of the absorbing layer to that of the original radiation. If we take the photosphere to be at a temperature of 10000°, this would give the temperature of the absorbing layer about 1500° lower. We may get another estimate of the ratio of the same temperatures by taking the ratio of the fourth roots of the two maxima of radiation. This ratio is 0.73, which would give a lower temperature of the absorbing layer. But all these calculations ought to be corrected for the absorption in our atmosphere, and I am perhaps trying to draw conclusions which the accuracy of the measurements do not as yet warrant. Sufficient has been said to show that important results may be expected, if Mr. Very's measurements are repeated and if a greater accuracy is obtained.



There is no reason to look for a different region in the Sun's surroundings for the cause of the observed diminution of radiation, than that which gives the Fraunhofer lines. The simplest supposition to make at present, and one consistent with our knowledge of spectra, is that the layer which gives the line absorption, absorbs also to some extent all wave-lengths extending from infra-red to violet, the diminution in the observed intensity of the solar radiation towards the edges of the disk would then simply be due to this absorption.

The principles which have been developed in this communication may find a wider application. Some observers have apparently been puzzled by the fact, that the radiation of the umbra of Sun-spots does not diminish as it nears the edge of the Sun in the same way as that of the luminous disk itself, but on the contrary remains nearly constant. Our tables show that in

the case of the solar disk only about half the radiation comes from the photosphere, the rest is made up by the radiation of the absorbing layer itself. If that absorption either by increased density or by greater thickness is increased four or five times, practically the whole of the radiation would come from the absorbing layer, and would be nearly constant for different portions of the solar disk. To see that this is the case it is only necessary to take the equation,

$$A = (I - F)\, e^{-kt} + F\ ,$$

and to substitute for e^{-kt} a value five times smaller than that given in Table II, or about 0.07. The whole radiation would then, even at the center, be sensibly equal to F.

MANCHESTER, ENGLAND,
November 19, 1902.

ON THE PROLONGATION OF SPECTRAL LINES.

By THEODORE LYMAN.

INVESTIGATORS who have worked with concave diffraction gratings cannot have failed to observe the faint but sharp prolongations of strong lines which occur in the spectra produced by these instruments.

The cause of the difference in length of certain lines when a prism spectroscope is used is well known. Sir Norman Lockyer long ago made use of the phenomenon of "long and short lines" in his study of the chemistry of the Sun. He pointed out that when the vertical slit of the collimator is illuminated by the image of a light source formed by a lens, some of the lines in the resulting spectrum are longer than others. This of course arises from the fact that the portion of the source which illuminates the center of the slit possesses some vibration frequencies which are wanting in those portions which come to focus at the top and bottom of the slit. When a concave grating is used, the astigmatism renders this phenomenon less striking.

The prolongations of strong lines which are referred to above present a different appearance, however, from those obtained with a prism spectroscope. They are, in fact, due to a different cause. As the author had never seen any explanation of the matter, it seemed that an investigation on the subject might prove of interest.

In making some adjustment of a concave grating of twenty-one feet radius the principal image of the slit came into view. It was at once noticed that this image was prolonged into two narrow streamers, one vertical and the other horizontal, each quite distinct and sharp. This suggested that the vertical continuation of the slit image and the vertical continuation of strong spectral lines were due to the same cause. Observations were accordingly made upon the principal image of the source. In order, however, to simplify the diffraction

PLATE XVI.

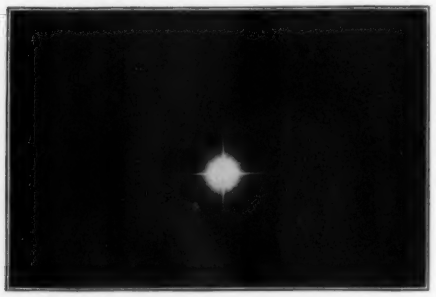


FIG. 1.

FIG. 2.

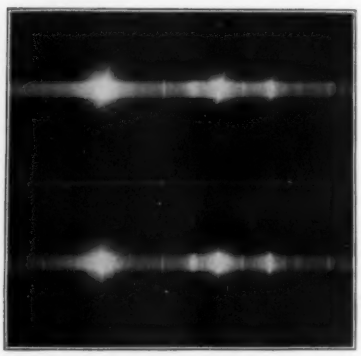


FIG. 3.

phenomena as much as possible a circular opening of about 0.01 cm diameter was substituted for the slit. In order to reduce spherical aberration to a minimum the light fell upon the grating at nearly normal incidence. In the final observations the polished surface was always covered. The first photograph was taken with the grating in its normal position and with all the ruled surface exposed. The result is an image of the pin hole accompanied by two sharp streamers, one vertical the other horizontal. The horizontal streamer is the longer and stronger of the two. The effect is shown in Plate XVI, Fig. 1.

If the ruled surface of the grating be protected by a screen having a rectangular aperture whose diagonal is less than the breadth of the ruled surface, the orientation of this aperture throws light upon the nature of the phenomenon. When the longest side of the opening is parallel to the longest side of the ruled surface, the image of the point source presents exactly the same appearance as when the whole grating surface is exposed. When, however, the aperture is set askew so that its sides make angles of 45° with the sides of the ruled surface, the effect on the image is striking. The result is shown in Fig. 2. The vertical streamer has revolved through 45° and the horizontal streamer has been broken into two parts. One of these parts has turned through 45° while the other and longer part has remained fixed. The streamers which accompany the image seem then to consist of two components, one a cross which turns as the sides which bound the ruled surface turned, the other a horizontal line or streamer which remains stationary. The idea that the movable cross is due to diffraction through a rectangular opening at once suggests itself. This theory may be further tested by covering the grating surface by a screen with a circular hole. The image of the source so obtained showed no vertical streamer, the horizontal line was still present though less intense. This result indicates that the explanation of the origin of the movable cross is the correct one.

When the point source was replaced by a slit the effects were of an exactly similar character.

It is easy to apply the results of these experiments to the vertical continuation of strong spectral lines. In order to demonstrate that the vertical streamers are due to the rectangular shape of the ruled surface, we have only to cover the surface with a screen whose rectangular aperture is set askew with respect to the direction of the lines of the grating. We obtain the effect shown in Fig. 3. Here the continuations of the strong lines have turned and broadened, though the lines themselves remain straight and sharp. The diffraction pattern from the rectangular grating aperture is no longer a cross with sharp vertical and horizontal arms. The cross has turned, and, since the aperture is no longer symmetrical with respect to the slit, the arms have broadened.

The horizontal band which does not turn as the aperture is rotated may be due to either of two causes or to a combination of them. The subordinate or secondary maxima which accompany every line in the spectrum furnish the first reason, irregularities in the grating ruling furnish the second. The author has shown that these irregularities may not only furnish a background, but, under favorable circumstances, may even produce sharp reproductions of real lines. The background or nearly continuous band can be noticed with almost every grating and can be best observed in that portion of the extreme ultra-violet where no real lines are obtained. Its intensity varies greatly with different gratings. In investigations where long exposures are necessary it often proves very inconvenient, for faint real lines are much obscured by its presence.

In many cases the horizontal band due to diffraction through a rectangular opening is much stronger and more troublesome than the band due to irregular ruling. In this case there is a remedy at hand. The ends of the ruled space may be covered with slanting pieces of black paper and the rectangular ruled space thus converted into a parallelogram. The effect of this arrangement is to revolve the horizontal streamer due to the shape of the opening, the

FIG. 4.

vertical streamer remaining fixed, with the result that the background of the spectrum is very materially cleared. The author can recommend this device to all those who investigate faint spectral lines, and to whom a clear field is a necessity.

JEFFERSON PHYSICAL LABORATORY,
Harvard University, November 1902.

Minor Contributions and Notes.

ON THE NUMBER OF STARS UPON A PHOTOGRAPHIC PLATE[1].

Since the foci for oblique rays if an objective can never lie in the same plane with axial focus, it must always happen that a part of the stars must form extra-focal images on a plane photographic plate. Only at that point where the plane of the plate cuts the geometrical position of the foci, will the image be focal, sharpest, and most intense; which will be the case, therefore, at the intersection of the plane of the plate with the focal surface. The greatest number of stars is, therefore, to be expected on the general average in this intersection curve, since the faintest stars are lost by defective focusing within it and without it. If the objective is not corrected for curvature of focal surface, the focal surface is a sphere with its center at the objective. If the plate is also perpendicular to the optical axis and pushed inside the axial focus, and if the center of the plate also coincides with the optical axis, the intersection curve will be a circle (focal circle), and the average number of stars will be solely a function of the distance from the center of the plate. If the plate is pushed along the optical axis, either inwards or outwards, the focal circle decreases or increases, and the number of stars varies accordingly. At what position of the plate shall we now obtain the greatest number of stars? This question may be solved on certain very general premises without knowledge of the complicated relations between light power and focusing as follows:

If we denote by δ the distance of any element of the plate from the focal sphere, we may assume that the optical effects depend only upon this δ, indifferently whether the element lies at a distance inside or outside the focal sphere, so long as the distances involved are small in comparison to the focal length. The number of stars on a plate can therefore be considered solely as a function of δ, which is related to the focal length f of the objective, and to the radius r of the focal circle, in the manner shown by the following simple geometrical considerations:

[1] Translated from a communication to the *K. Akademie in Wien*, Session of October 16, 1902, furnished by the author.

Outside the focal circle:

$$\delta_o = \frac{1}{2f}(r^2 - r_o^2) \ ,$$

Inside the focal circle:

$$\delta_i = \frac{1}{2f}(r_o^2 - r^2) \ .$$

This formula is sufficiently accurate up to a field of view of 4°. The number of stars is, therefore, an unknown function of δ or $F(r^2 - r_o^2)$. Hence the number of stars outside the focal circle becomes

$$A_o = 2\pi \int_{r_o}^{R} F(r^2 - r_o^2)\, r\, dr = \Phi(R^2 - r_o^2) - \Phi(o) \ ,$$

and inside the focal circle

$$A_i = 2\pi \int_{o}^{r_o} F(r_o^2 - r^2)\, r\, dr = \Phi(r_o^2) - \Phi(o) \ ,$$

where Φ is again an unknown function, and R is the distance from the center to the edge of the plate. The total number of stars is now

$$A = A_o + A_i = \Phi(R^2 - r_o^2) + \Phi(r_o^2) - 2\Phi(o) \ .$$

We see from the differential quotient

$$\frac{\partial A}{\partial r_o} = 2r_o[\Phi'(r_o^2) - \Phi'(R^2 - r_o^2)] \ ,$$

and from the obvious properties of the F- and Φ- functions that the number of stars is a maximum for the value $R^2 = 2r_o^2$ or $R^2\pi = 2r_o^2\pi$. We therefore obtain the greatest number of stars on a plate if we make the contents of the focal circle just half as great as the contents of the circular field of view to be photographed.

It follows from this rule that we must push in the plate from the axial focus by the amount

$$\delta_o = \frac{1}{2f} r_o^2 = \frac{R^2}{4f} = \frac{L2}{16f} \ ,$$

where L is the length of side of the square plate and f is the focal length. If we take $L = 165$ mm and $f = 3400$ mm for the Potsdam photographic refractor, the plate should be pushed in 0.47 mm in order to obtain the greatest possible number of stars. We see from the Potsdam plates for the *Carte celeste* that it was pushed in only 0.13 mm, whereby there has ensued on every plate an unnecessary loss, which may be estimated at least as 6 per cent. in the number of stars.

EGON VON OPPOLZER.

INNSBRUCK,
October 25, 1902.

REMARKS ON BIGELOW'S "ECLIPSE METEOROLOGY."

In the fourth chapter of his recently published work, Professor Bigelow also enters upon a discussion of the theory of Sun-spots for which I am sponsor. He does not appear to be quite favorably inclined to my theory, although I should not have expected that on the part of a meteorologist. There are two remarks in particular which force me to a reply.

1. In the section entitled "Remarks on the Computations by von Oppolzer and Fenyi," on p. 83, Mr. Bigelow writes: "Von Oppolzer, in his computation of the solar vertical temperature-gradient, uses $n = 27.6$, $(k-1):k = 1:3.5615$, $R = 420.552$, and finds $\frac{dt}{dh} = 18^{\circ}.96$ C. per 1000 m, which is 1463.4 times as large as my value ($0^{\circ}.0129562$ C.) given above. The difference comes from neglecting to evaluate the full value of n in the temperature and the gravity terms. For 1″ arc this gives a range of temperature 13667° C., which is an impossible result, and renders invalid the accompanying discussion." It is evidently clear that the discrepancy between his result and mine cannot be explained by any error of neglect, but only by one of principle. This result refers to the adiabatic decrease in temperature on the Sun, and by no means to the true decrease, as Mr. Bigelow seems to think, for this is at present unknown. The error is made by himself, and my computation is correct, and also my omissions are permissible. Mr. Bigelow believes simply that it is sufficient to be able to introduce in the formula for the terrestrial height of the barometer,

$$\log \frac{B_0}{B} = \frac{h - h_0}{K} = \frac{h - h_0}{RT},$$

the solar temperature 7535° C. in place of the temperature of the ground T, if there were air upon the Sun. He overlooks the fact that the barometric constant K, which is the height of the homogenous atmosphere, is inversely proportional to the surface gravity of the Sun. It is, however, instantly to be seen that with a greater force of gravity the height of the homogeneous atmosphere must under otherwise equal conditions become lower, and that the decrease in pressure must take place much more quickly, for if gravity is very slight the atmosphere will be high. Inasmuch as the surface gravity is 27.6 times greater on the Sun than on the Earth, our atmosphere would have on the Sun a homogeneous height of only 8000 m : 27.6 = 290 m.

According to Mr. Bigelow it would be of the same height, 8000 m. Such fundamental errors occur further in his formulæ. He believes that the adiabatic decrease in pressure can be obtained simply by introducing in the correction factor of the decrease of temperature of the ordinary barometric formula the adiabatic value, and then extends his computation up to the desired altitudes. He does not seem to be aware that the law of pressure can be obtained by a rigorous analytic process if the law of temperature is given. Thus on p. 83 he gives the integration of the equation

$$\frac{dp}{p} = -\frac{n}{RT}dh \quad \text{as} \quad \log\frac{p_0}{p} = -\frac{n}{RT}h \ ,$$

where n is a function of the temperature, and hence of h. Other remarkable statements are also found. If he means on p. 82 that the *large* barometer constant of 7296570 m which he incorrectly deduces for the Sun, has as a consequence a "rapid" decrease in pressure, then he is in error, for in this case it would be very slight. The smaller the constant, the more rapid is the decrease in pressure. I trust that these remarks will be sufficient to estimate the value of the considerations in this chapter, whereby, of course, I do not wish to extend my judgment to the remaining and surely valuable sections. Of course the tables for the decrease of pressure and temperature on the Sun are incorrect and valueless.

2. On p. 69 Mr. Bigelow writes with reference to my theory: "We cannot assume that the surface of the Sun would, like the Earth, appear dark if seen through a rift in the photosphere, and there is nothing to be gained on clearing the solar sky by descending currents." This also cannot apply in any way to my theory, since I expressly remarked that the upper layers of the photosphere are transparent, and partially protect the radiation of the lower layers. It would be a very unnatural assumption to regard the radiation as coming solely from the geometrical surface. An extreme reversal of temperature prevails over the nucleus of the Sun-spot, the existence of which I have doubtless sufficiently proven. Hence there lies over the nucleus an abnormally hot stratum which vaporizes the upper photospheric strata, consisting of the products of condensation (clouds), dissolves it, and hence produces clearness. *Above the nucleus of a spot there must therefore prevail a considerably less extinction than above the normal photosphere.* This also is shown in a convincing way by Wilson's measurements of the radiation of Sun-spots (*Proc. R. S.*, **55**, 248). After Langley had called

attention to the phenomenon, Wilson showed with a perfect chain of evidence that the radiation of a Sun-spot is less when it is at the Sun's limb than when it is at the center of the disk, on account of the absorption in the solar atmosphere, but that this decrease is *slower* toward the limb than over the normal photosphere. In connection with the fact that the level of the spot is depressed, this clearly shows that a less extinction or a greater clearness of the solar atmosphere prevails over the nucleus. If it was absolutely clear above the nucleus the radiation of a spot would be found to be the same at the center and at the edge of the disk. Wilson's observation, therefore, shows that there are still certain effects of cloudiness (extinctions) present over the spots.

These last considerations may serve to further support my anticyclonic theory of Sun-spots, and at the same time to allay the apparent difficulty arising from Wilson's observation on the one hand, and the depression of the level of the Sun-spots on the other, which has led several observers (Frost, Hale, ASTROPHYSICAL JOURNAL, **4**, 196; **6**, 366) to somewhat far-fetched explanations.

EGON VON OPPOLZER.

INNSBRUCK,
September 25, 1902.

MISCELLANEOUS RESULTS, NO. 1.[1]

THE circulars hitherto published have subserved their purpose in furnishing a more prompt publication of results than would be secured by waiting for the issue of the volume of Annals which would contain them. Much material has, however, accumulated which might fill several circulars. To avoid the consequent delay, some of the results are given below in a briefer form.

VISIBILITY OF *EROS*.

A letter lately received from Professor Bailey states that *Eros* was photographed with the Bruce Telescope on July 8, 1902. The telescope was made to follow a star by means of an eyepiece adjoining the plate, while a motion was given to the latter equal to the computed motion of *Eros*. The stars accordingly leave trails having a direction and length corresponding to the motion of *Eros*, while the planet appears as a minute dot, the entire light being concentrated for the whole time of exposure upon a few silver particles. A second photograph confirmed this observation. So far as known, the first visual

observation of *Eros*, since its recent conjunction with the Sun, was obtained by Professor H. A. Howe at the Denver Observatory on August 2, 1902, as announced in the *Harvard Bulletin* of August 5, 1902. It will be remembered that the first photograph of *Eros* after its conjunction in 1899 was also obtained with the Bruce telescope on April 28, 1900, by the method described above. It was first seen on June 6, 1900, at the Denver Observatory, and later at the Arcetri Observatory on June 30, 1900. In like manner, it was followed after the opposition of 1900, until March 12, 1901, at Arcetri, until June 17, 1901 at Denver, and until September 9, 1901, at Arequipa, with the Bruce telescope.

WILLIAMS' NEW *ALGOL* VARIABLE, 13.1902.

In order to identify the *Algol* variable, 13.1902, recently announced by Mr. A. S. Williams (*A. N.*, **159**, 309), its position was marked on several of the Draper Memorial photographs. On one of these, taken 1893, July $11^d\ 18^h\ 20^m$, G. M. T., the star appeared fainter than normal. On Mr. Williams' scale its magnitude was about 11.3, or a little nearer in brightness to his star *d*, than to his star *b*. It is also very nearly equal in brightness to the star which follows the variable about 45^s, north 6.′8. This would indicate that the time of minimum preceded or followed the time of taking the photograph by about three hours, assuming the light curve described by Mr. Williams. The formula he gives indicates for $E = -918$, a time of minimum J. D. 2,412,656.720 = 1893, July $11^d\ 17^h\ 17^m$, G. M. T., omitting his last three decimal places which appear to be indeterminate.

This photograph, therefore, gives a correction to the ephemeris of $+4^h$ or -2^h. An examination of the other plates of this region would distinguish between these two values, and determine the correct one with much greater accuracy. Probably we have at least one hundred photographs of this region, although the star may be too faint to appear on a portion of them. On one-tenth of the photographs the star should be below its normal brightness. A precise correction to the ephemeris can be found from each of these, if not too near maximum or minimum, when the photographic light curve has been found, as can now be readily done. Unfortunately, the pressure of other work will probably prevent the continuation of this research at present. As it is, the observation given above lengthens the period of observation from less than ten months to about nine years, and increases the

[1] *Harvard College Observatory Circular* No. 66.

accuracy with which the period can now be determined in nearly the same proportion.

EARLY OBSERVATIONS OF *NOVA PERSEI*, NO. 2.

As the *Nova* is diminishing in light it becomes necessary to use comparison stars so faint that they are not contained in the *Harvard Annals*, Volume XLV or other photometric catalogues. A sequence has accordingly been selected which is given in Table I. The designation is followed by the number, number of grades, difference in right ascension, and difference of declination from the *Nova*, taken from *Hagen's Second Catalogue*. The sixth column gives the photometric magnitude, found for the stars *b* to *f* from comparisons by Professor Wendell with star *a*. Constructing a curve with coördinates taken from the second and sixth columns, we have the means of converting the estimates of Hagen into magnitudes on the photometric scale. The seventh column gives the residual found by subtracting the magnitude given in the sixth column from the Hagen magnitude thus formed. Table II gives the photometric magnitude corresponding to various values of Hagen's grades. The fainter magnitudes found by interpolation are somewhat uncertain.

TABLE I.

SEQUENCE OF COMPARISON STARS.

Des.	H.	G.	Δα	Δδ	Magn.	H.-P.	Des.	H.	Gr.	Δα	Δδ	Magn.	H.-P.
			s	′						s	′		
a	20	67	−21	+4.8	9.10	+.03	*h*	65	178	+42	+4.8	..	..
b	32	84	+43	+8.7	9.71	+.19	*k*	66	179	+33	−4.1	..	..
c	34	92	−65	+3.4	10.57	−.32	*l*	68	185	+34	+1.5	..	..
	42	111	+35	+0.2	11.03	+.01	*m*	74	199	+21	+1.6	..	..
e	46	133	−82	+6.3	11.94	−.08	*n*	77	209	+13	+1.0	..	..
f	49	145	−24	−1.5	12.23	+.03	*o*	80	..	− 3	−0.5	..	..
g	56	161	−17	+2.4	..	..							

Table II furnishes the means of determining the limiting magnitude below which the *Nova* must have been before its discovery. The last photograph taken before it was found on February 21, 1902, was obtained by Mr. A. S. Williams on February 20. It is reproduced in *Knowledge*, **24**, 152. All stars included in the region covered by the print, and brighter than *Hagen* No. 42, are shown except Nos. 29, 34, 36, 38, and 39. The absence of 39 is perhaps due to the proximity of

No. 28. Stars of the magnitude 10.9 are therefore shown, and we may, hence, infer that the *Nova* was below this limit on February 20. Several defects appear on this print which cannot be distinguished from stars. The next photograph preceding this was obtained at this Observatory, and is shown in Fig. 1 of *Circular* No. 57. This print shows all stars brighter than No. 49, except No. 39. The *Nova* must therefore have been fainter than the magnitude 12.0 on that date.

TABLE II.

RELATION OF HAGEN'S GRADES TO MAGNITUDES.

Grades	Magnitudes	Grades	Magnitudes	Grades	Magnitudes	Grades	Magnitudes
0	6.56	50	8.39	100	10.59	150	12.42
10	6.90	60	8.82	110	11.00	160	12.72
20	7.25	70	9.27	120	11.39	170	13.00
30	7.61	80	9.72	130	11.76	180	13.25
40	7.99	90	10.16	140	12.10	190	13.48

About a year ago, Father Zwack of the Georgetown College Observatory called my attention to a faint star which appeared on one of our early photographs so near the position of the *Nova* that careful measurements were required to determine whether the positions are identical or not. A measurement of several of the early plates gave the magnitudes of this object on October 26, December 2, 1890, January 20, March 11, December 10, 1891, January 25, 1893, October 11, 1894, October 17, 1897, and March 7, 1900, as 12.95, 13.37 $<$13.7, $<$13.4, $<$13.7, 14.06, 13.15, 13.46, and 13.36, respectively. The images of this star on the photographs taken in 1890 are somewhat uncertain, but a comparison of the photographs taken in 1893 and 1894 leaves little doubt of the variability of this object. Measurements of its position were made on the last three photographs mentioned above. Taking the position in which the *Nova* appeared as an origin, the values of x for this star are $-1''.6$, $+1''.2$, and $+0''.2$, those of y, $+1''.3$, $-1''.0$, and $-0''.2$, respectively. These measures are based on the positions of the stars Nos. 20, 56, 77, 78, 79, 80, and 81, determined micrometrically at the Lick Observatory.

The object announced by M. S. Blajko, *A. N.*, **157**, 193, which according to his measures, followed the *Nova* $0^s.31$, south $7''$, on January 30, 1899, is doubtless identical with this object. Hagen's double

star, No. 69, whose position is 40″ distant from the *Nova* does not appear on any of these photographs, although much fainter stars are shown.

We may therefore conclude that a star whose light varied from the thirteenth to the fourteenth magnitude was visible for several years within one or two seconds of arc of the *Nova*, the difference in position being less than the errors of measurement.

EDWARD C. PICKERING.

October 31, 1902.

COMMENT ON THE EARLY HARVARD PHOTOGRAPHS OF *NOVA PERSEI.*

IN *Harvard Circular* No. 66, Professor E. C. Pickering calls attention to a small star of the thirteenth or fourteenth magnitude, close to the present position of *Nova Persei*, which appears on a number of plates taken between October 26, 1890, and March 7, 1900. The photographs show this star to have been variable, ranging in light from the thirteenth to the fourteenth magnitude.

His measures of the position of the small star with respect to the place of the *Nova*, on three plates, viz:

$$\Delta\alpha \; -1''.6, \; +1''.2, \; +0''.2 \; ;$$
$$\Delta\delta \; +1.3, \; -1.0, \; -0.2 \; ;$$

give in the mean

$$\Delta\alpha \; -0''.1$$
$$\Delta\delta \quad 0''.0 \; .$$

The measures above would indicate an exact identity of this small star with the *Nova*, as is implied in the last paragraph of the *Circular*. The fact that the star was variable would further strengthen this supposition. If we accept this, and it would appear to be almost certain, it would appear that the *Nova* formerly existed as a very small variable star not brighter than the thirteenth magnitude, whose variation was about one magnitude.

When the *Nova* has faded somewhat further, it will be possible to decide the question of identity, for the large telescopes will readily show if there are two stars at this place. The *Nova* is fading at the rate of about 0.2 magnitude per month and is now of about the tenth magnitude. Hence at this rate, by the first of the year 1904 the star will have reached the normal magnitude it had before the great outburst of light in 1901. It will probably not sink beyond the thirteenth

or fourteenth magnitude, when it will be interesting to see if it becomes variable again.

It is possible that the *Novæ* in expiring return to their original condition, and that such stars as *Nova Cygni* of 1876 existed previously as stars of the magnitude they now have. *Nova Coronae* of 1866 is the only one of these remarkable stars which was positively known previous to its outburst. In fading it went back to its original condition of brightness. Photographic records will in the future be of the utmost value in settling this question.

Professor Pickering speaks of the star announced by M. Ceraski in *A. N.*, **157**, 193, as probably being the same as the small star shown on the Harvard photographs. This seems improbable unless we allow a very large error in the position assigned by M. Ceraski. His star was of the twelfth magnitude and was $0^s.31$ ($3''.4$) following and $7''$ south of the place of the *Nova Persei*. This would put it in Position Angle $154°.1$, and Distance $7''.8$. I have looked for this star on a number of occasions since the *Nova* has become faint but have not seen it. It does not appear to have been shown on the plates of this region measured by Professor Perrine (*L. O. Bulletin* No. 23). Its possible identity with the Harvard star would be very interesting and can readily be proved if M. Ceraski will remeasure its position on the 1899 plate with reference to any of the stars used by Aitken (*L. O. Bulletin* No. 18), or by me in the measures in *A. N.*, **159**, 49, 1902.

Should the small variable star shown on the Harvard photographs really prove to be the *Nova* the discovery will be only another of the many evidences of the extremely great value of the Harvard College Observatory photographs, which seem to be a veritable mine of the richest value, the thorough working of which would lead to many important discoveries yet unknown but which are there faithfully recorded on the photographic plates.

E. E. Barnard.

Yerkes Observatory
November 18, 1902.

NOTE ON REFLECTORS FOR ECLIPSE WORK.

During the course of some investigations in which I had occasion to use reflected light, I observed incidentally that silver begins rapidly to lose its reflecting power even in the visible violet, and sinks nearly to zero, for normal incidence, in the region of λ 3200, beyond which it again becomes a reflector, but only a very poor one in the rest of the photographic region.

Further investigations along this line were abandoned as soon as I became aware, through recent publications,[1] of the excellent work of Hagen and Rubens on metallic reflection. But while I have nothing to add, beyond the fact that our results with silver reflectors, obtained by different methods, are in perfect agreement, I wish to direct specific attention to some practical uses in eclipse work, and other places where economy of light is important, that should be made of their observations.

The following table, condensed from a much longer one in the original paper, contains the results of most value in this connection.

PRECENTAGE REFLECTED OF NORMALLY INCIDENT LIGHT.

For λ =	2510	2880	3050	3160	3260	3380	3570	3850	4200	4500	5000	5500	6000	6500	7000
By silver (fresh)	34.1	21.2	9.1	4.2	14.6	55.5	74.5	81.4	86.6	90.5	91.3	92.7	92.6	93.5	94.6
By speculum metal (62.2 *Cu* +31.8 *Sn*)	29.9	37.7	41.7	....	....	....	51.0	53.1	56.4	60.0	63.2	64.0	64.3	65.6	66.8
By Schröder's alloy (66 *Cu* +22 *Sn* +12 *Zn*)	40.1	48.4	49.8	....	54.3	....	56.6	60.0	62.2	62.6	62.5	63.4	64.2	65.1	67.2
By Mach's magnalium (69 *Al* +31 *Mg*)	67.0	70.6	72.2	....	75.5	....	81.2	83.9	83.3	83.4	83.3	82.7	83.0	82.1	83.3

The atmosphere, as is generally familiar, absorbs ultra-violet light to such an extent that but little is known of the solar spectrum beyond λ 3000, while even the light of considerably longer wave-lengths is still feeble. It therefore is a matter of the utmost importance in photographing this part of the spectrum during a total eclipse, when the exposures have to be very short, to economize its light as far as practicable.

In accordance with the above results I would suggest that, in eclipse work, spectrographs be used, if possible, without the aid of reflectors of any kind, and that in those cases where, because of the type of spectrograph, or for any other reason, a reflector is essential, it be selected according to the portion of the spectrum to be studied. For λ greater than 4000 a thick film of silver freshly deposited on glass is the best reflector known, while for much of the valuable ultra-violet it is all but worthless. However, where silver fails, Mach's magnalium succeeds, and seems to be the best reflector of ultra-violet light so far examined.

Evidently, therefore, when spectrographs, used in eclipse, or any other work, are supplied by reflected light, it would be best to use silver reflectors for wave-lengths greater than λ 4000, and Mach's

[1] *Annalen der Physik*, **8**, 1–21, and 432–454, 1902.

magnalium, unless it should give too much trouble due to tarnishing, for the ultra-violet.

In those cases where much severe exposure is unavoidable, a Schröder reflector would probably be the best, since it is said to be but slightly affected by prolonged exposure to all sorts of weather, and besides, its coefficient of reflection, while much less than that of Mach's magnalium, is still distinctly better than that of speculum metal in the ultra-violet, and quite its equal for longer wave-lengths.

W. J. HUMPHREYS.

UNIVERSITY OF VIRGINIA,
Charlottesville, November 28, 1902.

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The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed.

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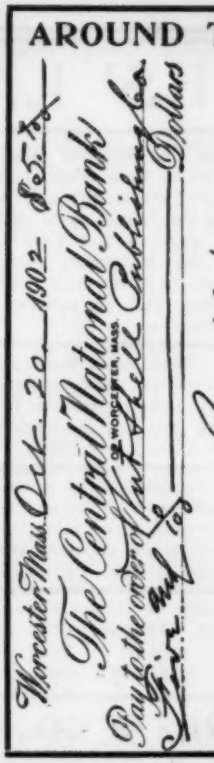
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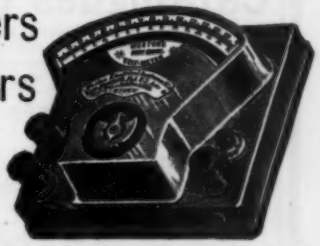

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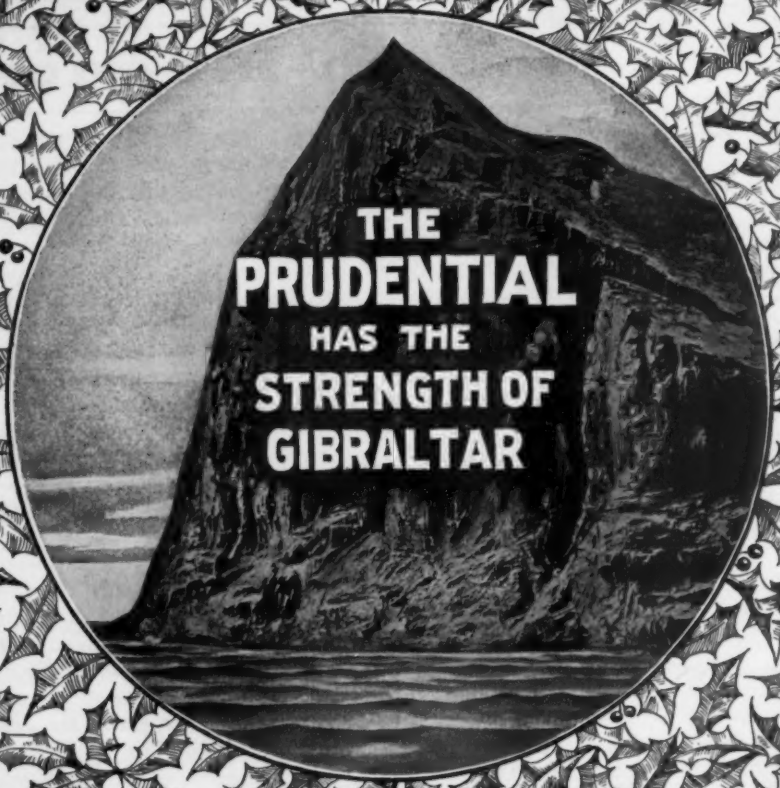

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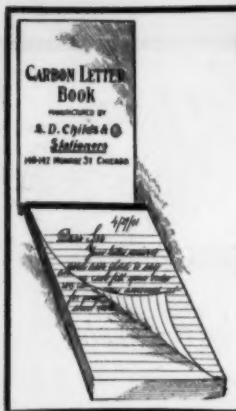

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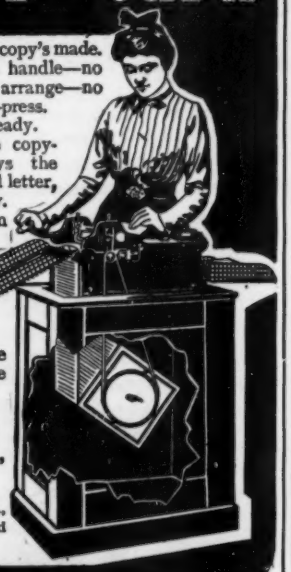

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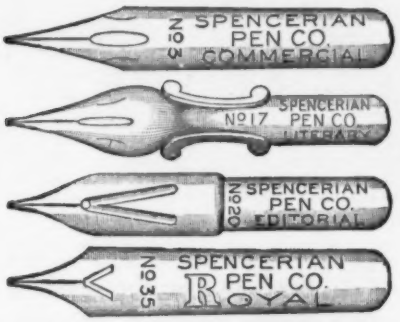
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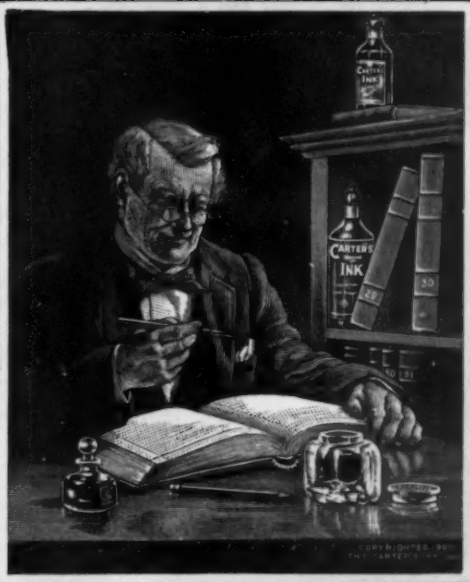

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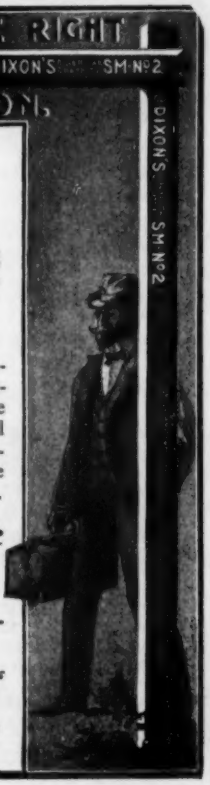

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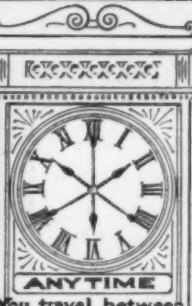

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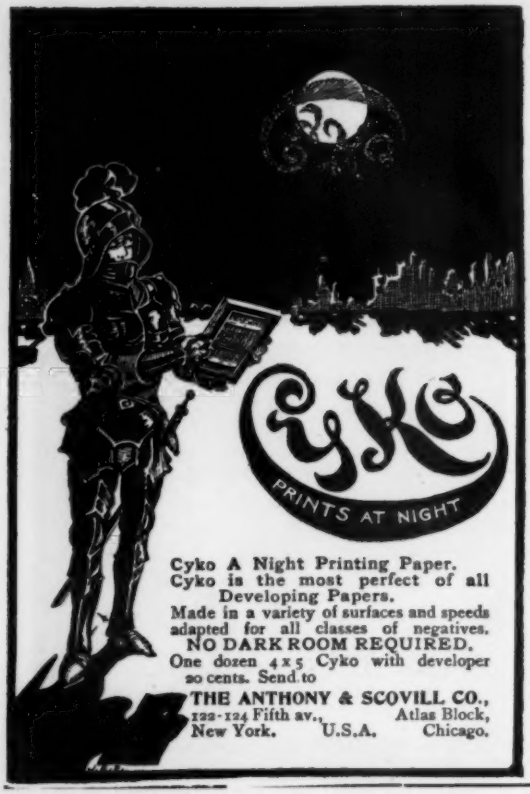

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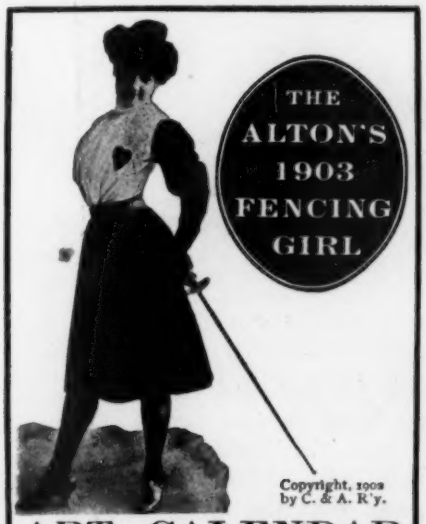

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